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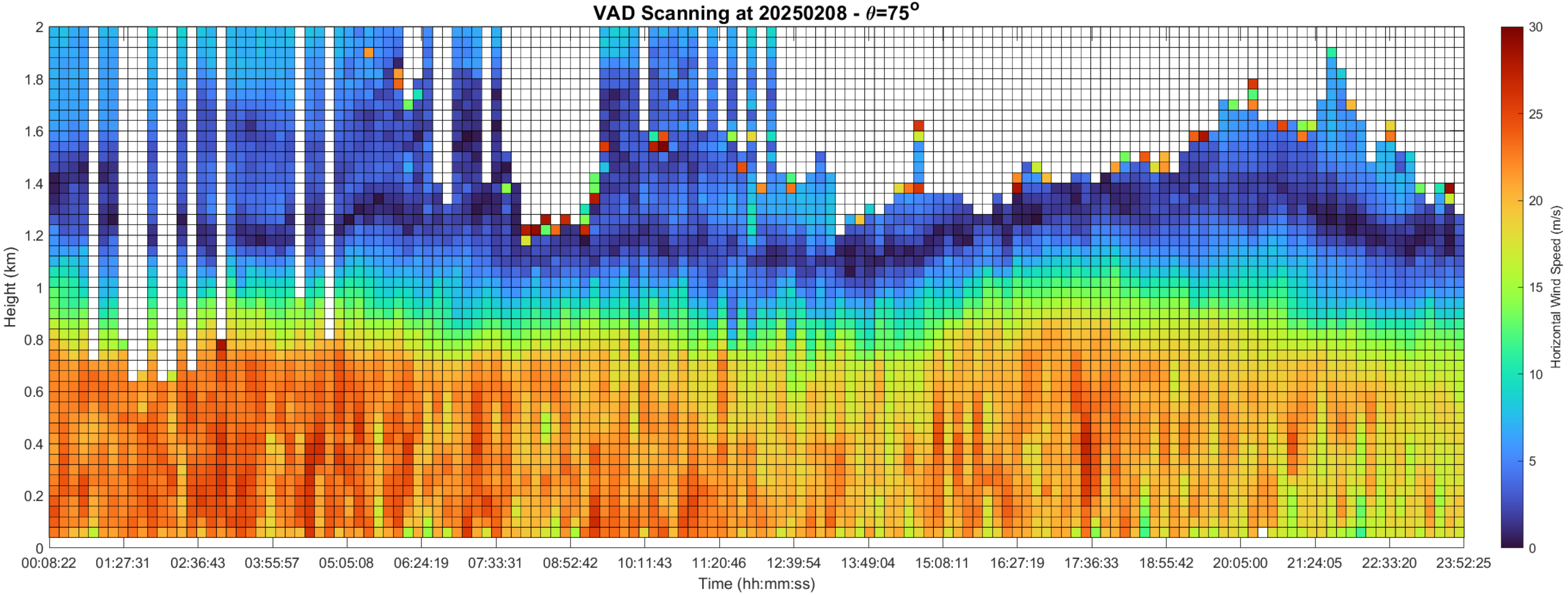
Doppler Lidar Data Analysis with a Viscous Flow Theory

苗君易、陳喻歆、 陳盈如、李承佑、商清妍、 紀惠珊

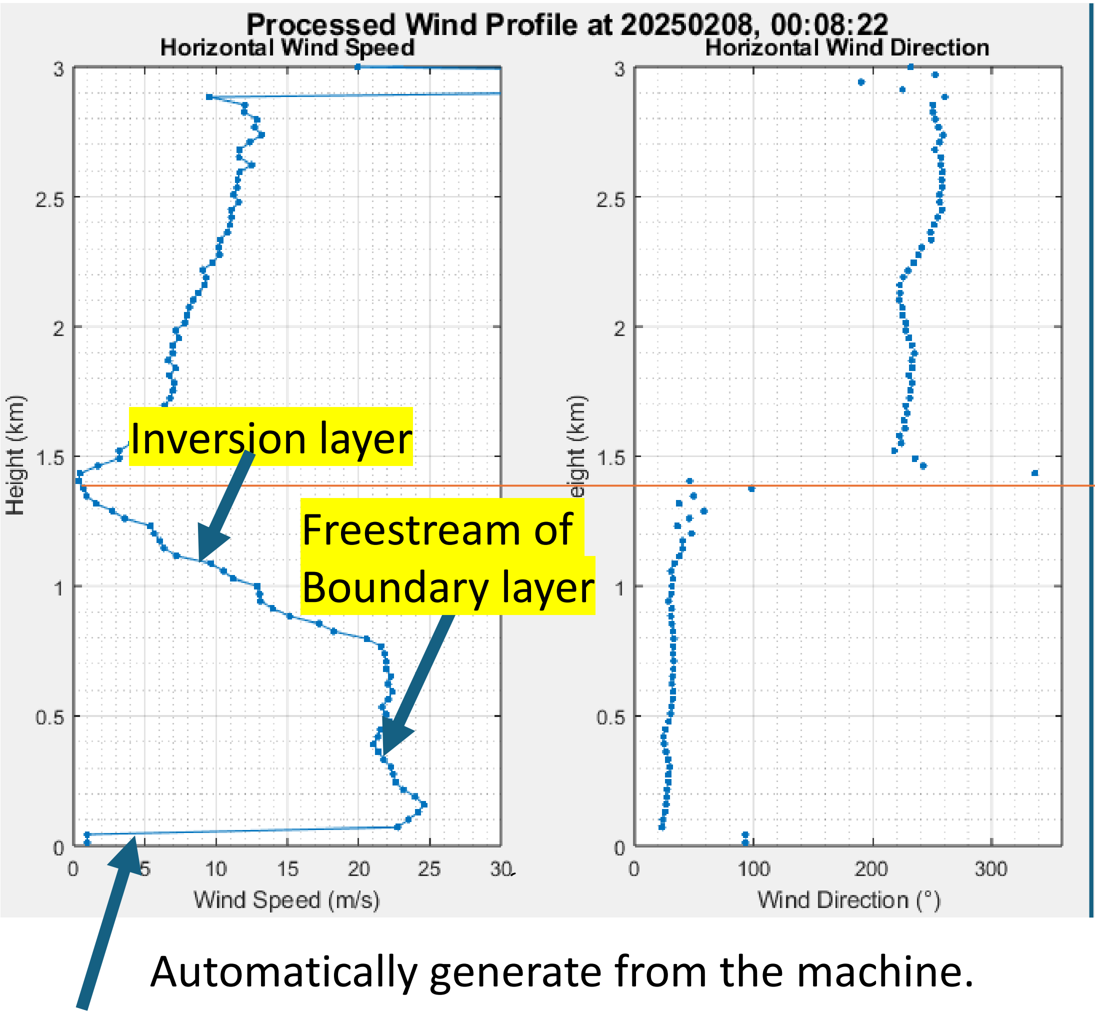
國立成功大學 航太系

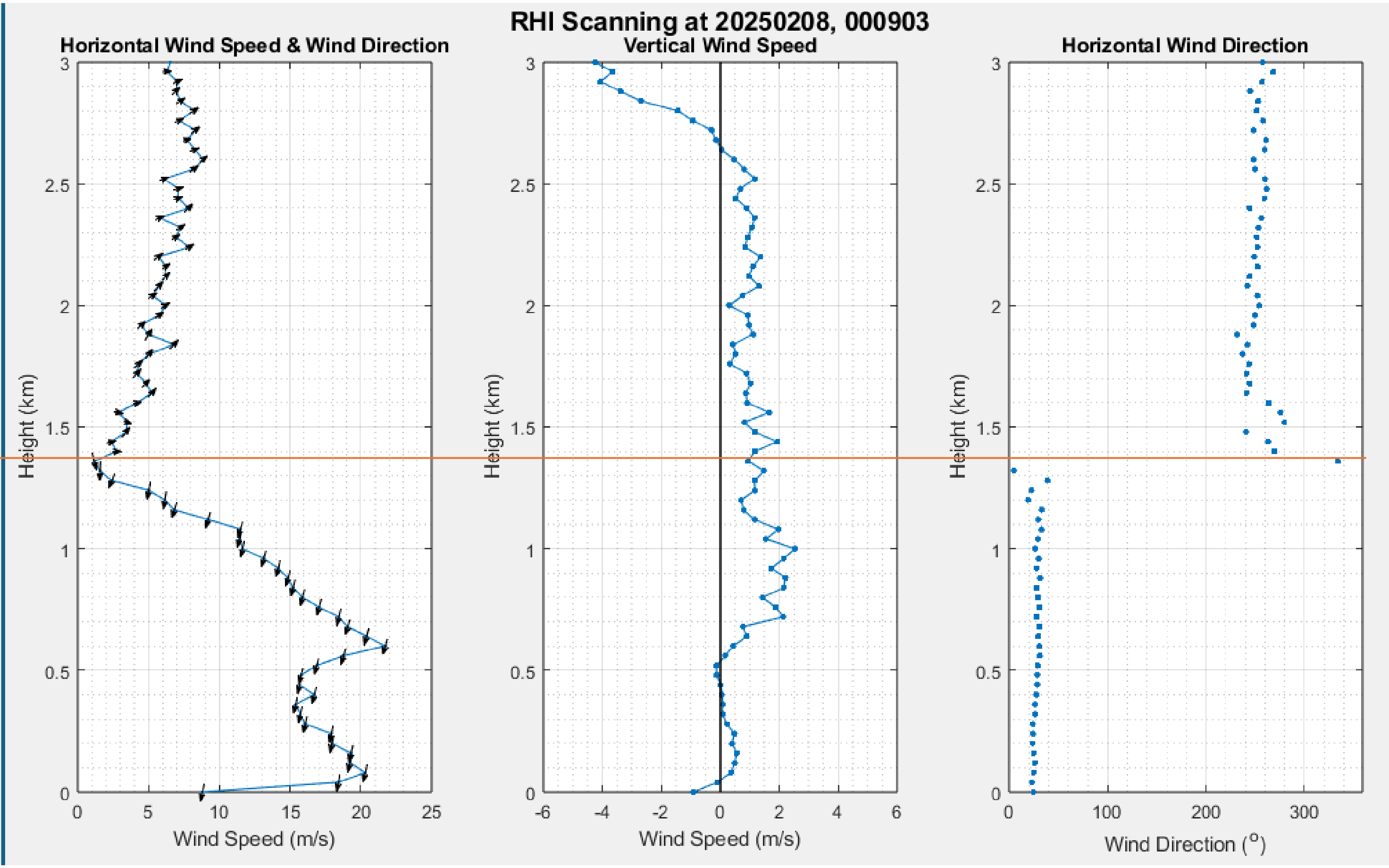
Motivation

- Doppler lidar measurements (20250208) provided the information concerning the atmospheric boundary layer at Taichung harbor subjected to strong north-east wind



Sampled data (20250208, 000822 and 000903)





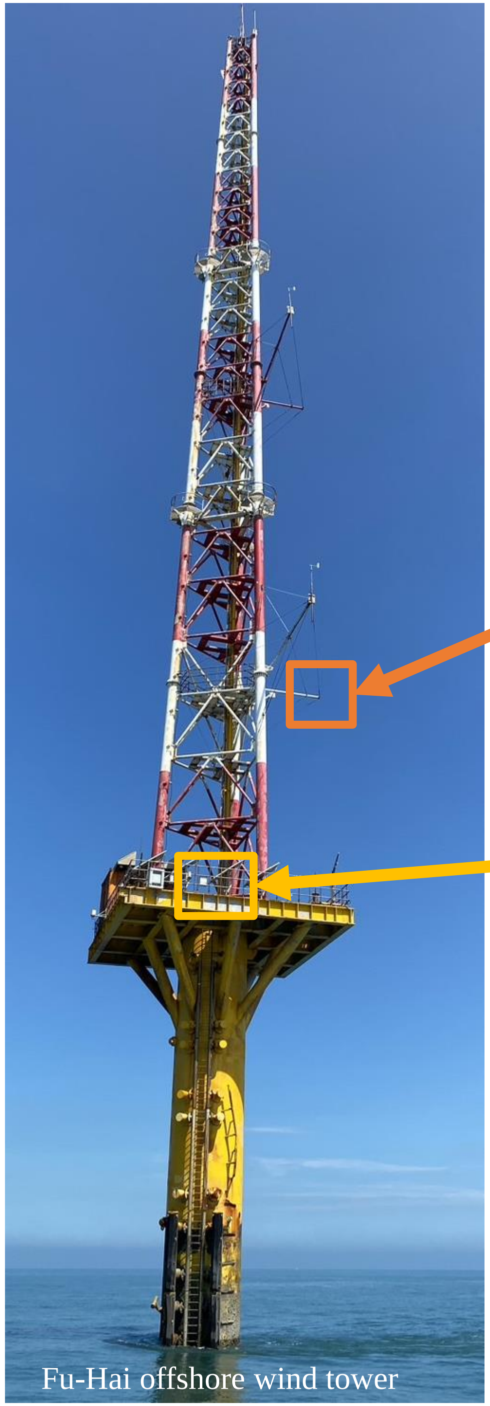
Automatically generate from the machine.

surface layer

Calculate the wind profile from the RHI scanning.
Azimuth angle: 0~170°/ each 10°
Elevation angle: 0~180° / each 10°

Offshore met tower provided the wind and wave data

- 永傳能源公司的福海海氣象觀測塔
- 彰化芳苑鄉外海8公里處
- Tower height: 86 m


Fu-Hai offshore wind tower

Ultrasonic meter:

Observation items: Wind speed(u, v, w, and total wind speed; m/s), wind direction

Sampling rate: 2 Hz

Instrument height: 30 m

Observation frequency: Once every 20 minutes (three times per hour), each lasting 512 seconds

Water level gauges:

Observation items: Water level variation(m), wave height(m), period(s), wave direction

Sampling rate: 2 Hz

Instrument height: 25 m

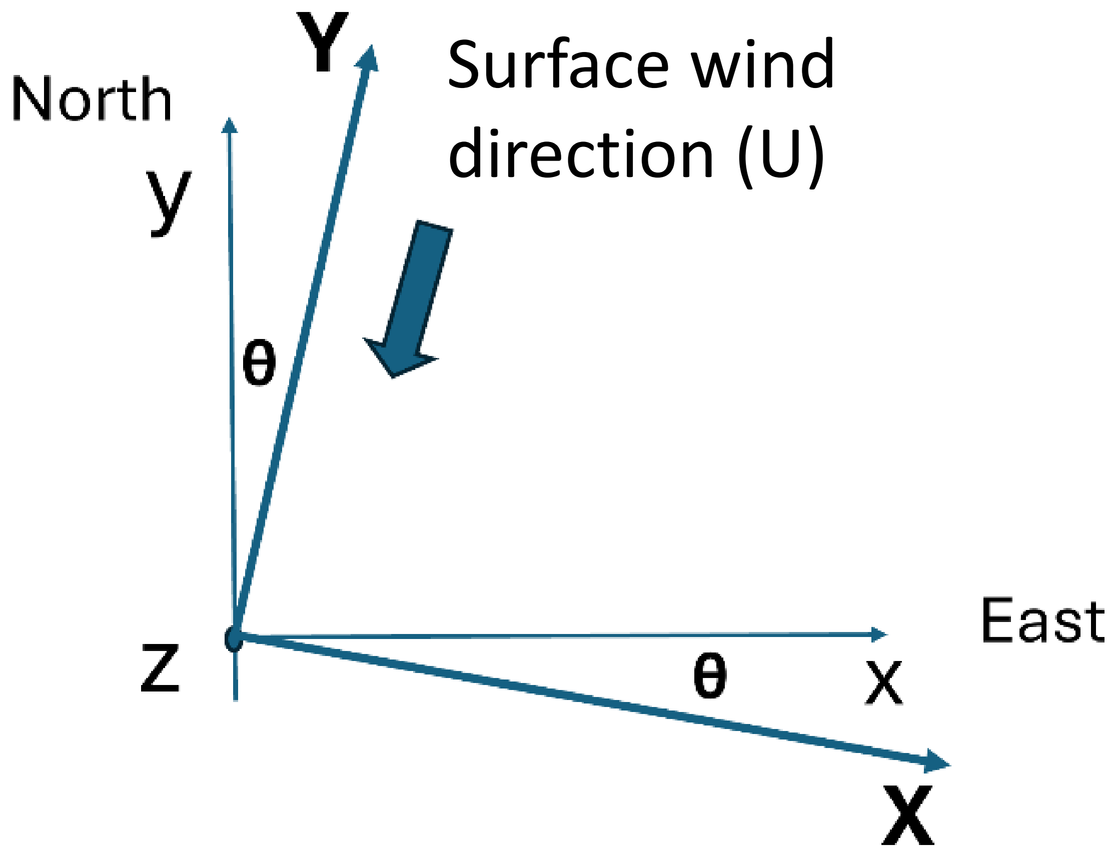
Observation frequency: Once every 20 minutes (three times per hour), each lasting 512 seconds

Objectives

-To conduct a viscous flow analysis on the atmospheric boundary layer offshore Taichung harbor, with emphasis on the surface roughness effect

-To conduct a comparison of k_s and z_0, which are the two quantities separately characterizing the aerodynamic roughness

Coordinate systems



u: wind speed in the x direction

v: wind speed in the y direction

U: the surface wind speed in the direction θ from North

The two-dimensional, time-mean momentum equations describing the atmospheric boundary layer are shown below with the rectangular coordinate system defined. (Pedlosky, 1979)

$$\frac{Du}{Dt} = fv - \frac{1}{\rho}\frac{\partial P}{\partial x} + (\text{Reynolds stress}) + \nu \frac{\partial^2 u}{\partial z^2} \quad (1)$$

$$\frac{Dv}{Dt} = -fu - \frac{1}{\rho}\frac{\partial P}{\partial y} + (\text{Reynolds stress}) + \nu \frac{\partial^2 v}{\partial z^2} \quad (2)$$

Pedlosky, J., *Geophysical Fluid Dynamics,* in Chapter 4: Friction and viscous flow. pp. 172-174, Springer-Verlag, 1979.

In the freestream of the boundary layer, the momentum equations (1) and (2) can be simplified by ignoring the contributions of Reynolds stresses and viscous stresses

$$\left.\frac{Du}{Dt}\right)_{freestream} = fv - \frac{1}{\rho}\frac{\partial P}{\partial x} \qquad (3)$$

$$\left.\frac{Dv}{Dt}\right)_{freestream} = -fu - \frac{1}{\rho}\frac{\partial P}{\partial y} \qquad (4)$$

On the other hand, at z=0, owing to the no-slip boundary condition: u=v=0. The equations are simplified and shown below. Namely, the pressure gradient term is balanced by the viscous force term.

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left.\frac{\partial^2 u}{\partial^2 z}\right)_{z=0} \qquad (5)$$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left.\frac{\partial^2 v}{\partial^2 z}\right)_{z=0} \qquad (6)$$

Along the wind direction, the total wind speed is denoted as U: $U=(u^2+v^2)^{1/2}$. Therefore, the momentum equation can be written below.

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial Y} + \nu \frac{\partial^2 U}{\partial z^2}\Bigg)_{z=0} \quad (7)$$

According to the boundary layer theory

$$\frac{\partial p}{\partial Y}\Bigg)_{z=0} = \frac{\partial p}{\partial Y}\Bigg)_{freestream}$$

In the freestream

$$\frac{\partial p}{\partial Y}\cos\theta = \frac{\partial p}{\partial y}$$

$$\frac{\partial p}{\partial Y}\sin\theta = \frac{\partial p}{\partial X}$$

Methodology

To evaluate the viscous force at ocean surface, which is balanced by the pressure gradient

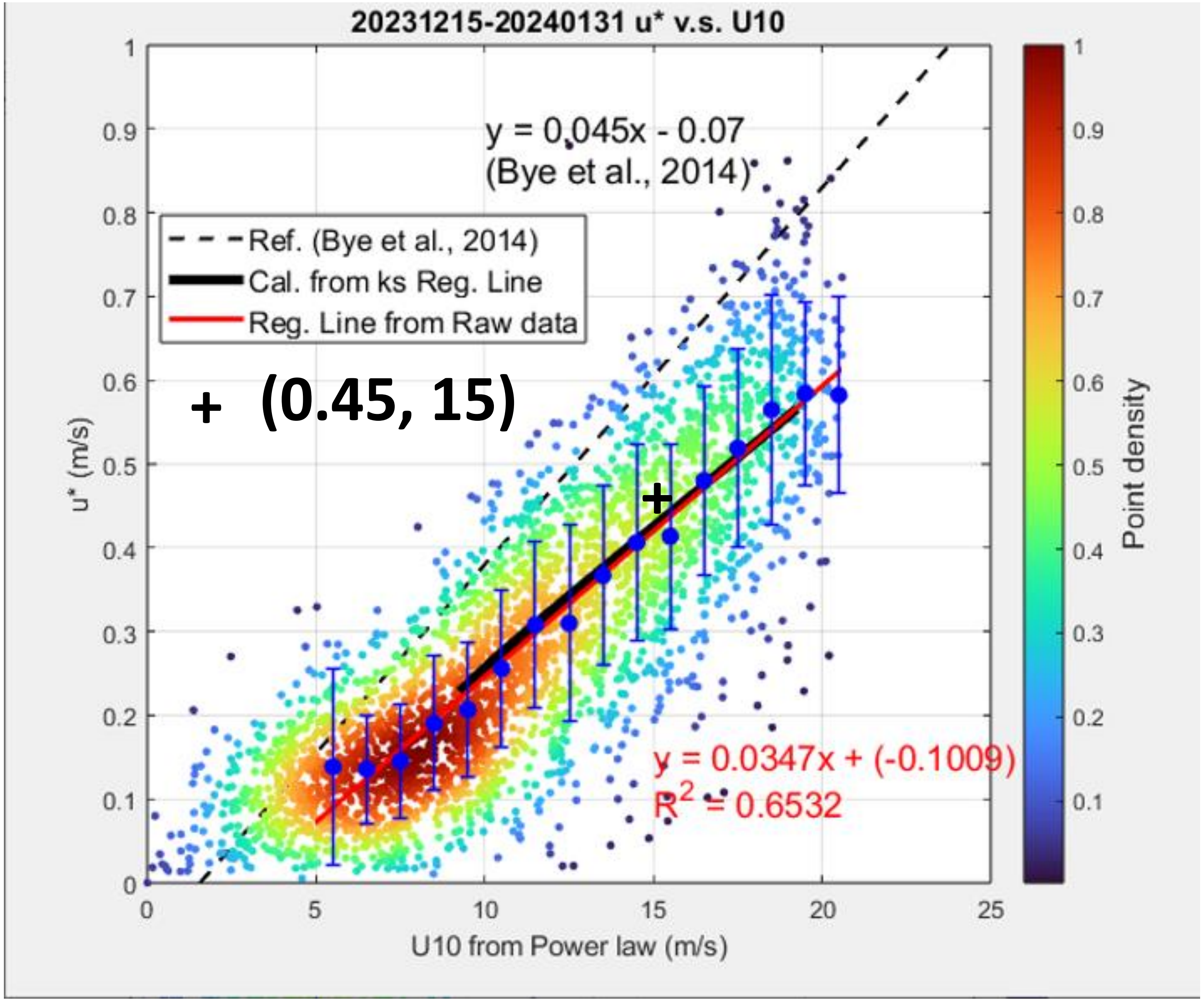
To compare Coriolis force and Pressure gradient in the freestream, which is equivalent to the pressure gradient at the ocean surface

Input:
-Friction velocity reduced from the ultrasonic anemometer
-Log velocity profile for a fully-roughened boundary layer (Schlichting, 1968) to find k_s
-Charnock relation (1955) to find z_0

Discussion:
-To evaluate the pressure gradient at the location of the met tower
-To compare k_s and z_0 within the uncertainty range of friction velocity reduced from ultrasonic meter measurements

Consider a case at
+: (0.45, 15),
u*=0.45 m/s
U_{10}=15 m/s

Uncertainty included:
u*=0.45 ± 0.05 m/s
U_{10}=15 m/s



Fully-roughened turbulent boundary layer

$$U_{10}^{+} = \frac{1}{0.4} \ln\left(\frac{10}{k_s}\right) + 8.5$$

(for $u^* = 0.45\ m/s$)

$$\frac{15\ m/s}{0.45\ m/s} = \frac{1}{0.4} \ln\left(\frac{10}{k_s}\right) + 8.5$$

$$(33.3 - 8.5) \times 0.4 = \ln\left(\frac{10}{k_s}\right);\quad 2 \times 10^{4} = 10/k_s;\quad k_s = 0.5 \times 10^{-3}\ (m)$$

(for $u^* = 0.5\ m/s$)

$$\frac{15\ m/s}{0.5\ m/s} = \frac{1}{0.4} \ln\left(\frac{10}{k_s}\right) + 8.5$$

$$(30 - 8.5) \times 0.4 = \ln\left(\frac{10}{k_s}\right);\quad 5.4 \times 10^{3} = 10/k_s;\quad k_s = 1.85 \times 10^{-3}\ (m)$$

(for $u^* = 0.4\ m/s$)

$$\frac{15\ m/s}{0.4\ m/s} = \frac{1}{0.4} \ln\left(\frac{10}{k_s}\right) + 8.5$$

$$(37.5 - 8.5) \times 0.4 = \ln\left(\frac{10}{k_s}\right);\quad 1.1 \times 10^{6} = 10/k_s;\quad k_s \sim 10^{-5}\ (m)$$

Discussion :

- In $U_{10}^{+} = \frac{1}{0.4} \ln\left(\frac{10}{k_s}\right) + 8.5$,

  k_s is very sensitive to u^* in U^+

  (i) $u^* = 0.45$ m/s ; $k_s \sim 0.5 \times 10^{-3}$ m

  (ii) $u^* = 0.5$ m/s ; $k_s \sim 1.85 \times 10^{-3}$ m

  (iii) $u^* = 0.4$ m/s ; $k_s \sim 10^{-5}$ m

- Using Charnock relation (Pena & Gryning, 2008)

$$\frac{z_0 g}{u^{*2}} = \text{const } (0.012)$$

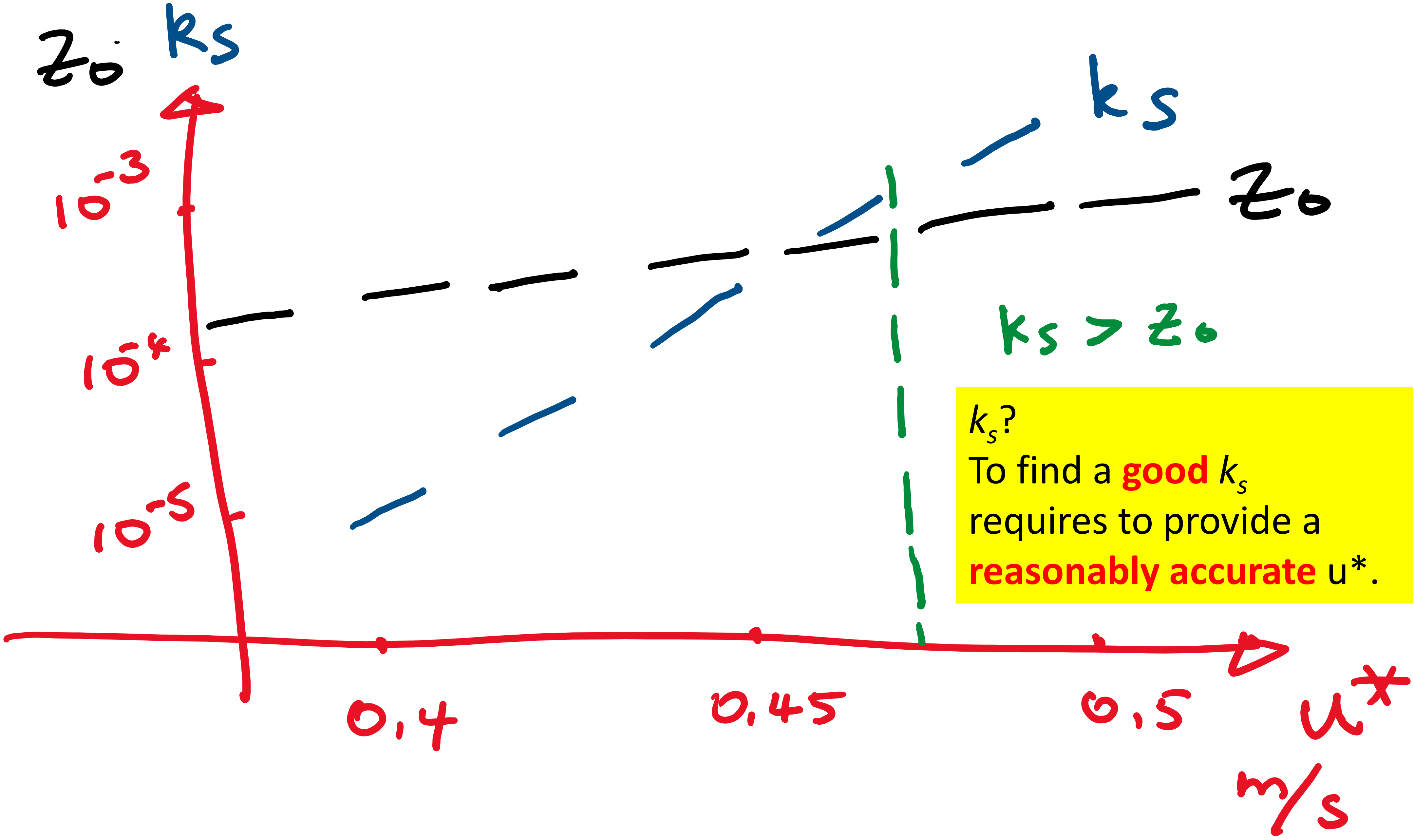
z_0 is less sensitive to u^*, compared to k_s

(i) $u^* = 0.45 \, m/s$; $z_0 = 0.24 \times 10^{-3} \, m$

(ii) $u^* = 0.5 \, m/s$; $z_0 = 0.3 \times 10^{-3} \, m$

(iii) $u^* = 0.4 \, m/s$; $z_0 = 0.19 \times 10^{-3} \, m$

Alfredo Pena and Sven-Erik Gryning, "Charnock's roughness length model andnon-dimensional wind profiles over the sea", Boundary-Layer Meteorology, 2008, 128:191-203.



k_s?

To find a **good** k_s requires to provide a **reasonably accurate** u*.

The wall function (Spalding 1961)

$U^+ = 2.5 \ln z^+ + 8.5$ (in the log region)

$(U^+ - 8.5)/2.5 = \ln z^+$; $z^+ = e^{(U^+ - 8.5)/2.5}$

$= e^{0.4U^+} e^{-\frac{8.5}{2.5}} = 0.033 e^{0.4U^+}$

$U^+ = z^+$ (at the wall)

Therefore, near the wall the velocity profile can be approximated by

$$z^+ = U^+ + 0.033\left(e^{0.4U^+} - 1 - 0.4U^+\right) \quad (10)$$

← correction term

$$(0.4U^+)^2/2! + (0.4U^+)^3/3! + (0.4U^+)^4/4! + \cdots$$

To find the viscous force at the ocean surface

$$z^+ = U^+ + 0.033\,(0.4U^+)^2/2 \; ; \quad z^+ \to 0 \text{ or } U^+ \to 0$$

$$dz^+ = dU^+ + 0.00528\,U^+\,dU^+ \Rightarrow dz^+ = dU^+(1+0.00528\,U^+)$$

$$\frac{dU^+}{dz^+} = \frac{1}{(1+0.00528\,U^+)}$$

$$\frac{d^2U^+}{dz^{+2}} = -\frac{0.00528\,dU^+/dz^+}{(1+0.00528\,U^+)^2} = -\frac{0.00528\left(\frac{1}{1+0.00528\,U^+}\right)}{(1+0.00528\,U^+)^2}$$

For $z^+ = 0$, $U^+ = 0$

$$\left.\frac{d^2U^+}{dz^{+2}}\right)_{z^+=0} = -0.00528\,; \quad \frac{d^2(U/u^*)}{d(z/k_s)^2} = -0.00528, \quad \left.\frac{d^2U}{dz^2}\right)_{z=0} = -0.00528\,\frac{u^*}{k_s^2}$$

$$\left.\frac{d^2U}{dz^2}\right)_{z=0} = -0.00528\,\frac{u^*}{k_s^2} = -0.00528\left(\frac{0.45}{(0.5\times10^{-3})^2}\right)$$

$$= -0.00528\left(0.45/0.25\right)\times10^6$$

$$\left.\nu\,\frac{d^2U}{dz^2}\right)_{z=0} = 16\times10^{-6}\left(-0.00528\times1.8\times10^6\right)$$

$$= -0.152\left(m/s^2\right) \qquad \left(\text{for } u^* = 0.45\,m/s\right)$$

$$\left.\frac{1}{\rho}\,\frac{\partial P}{\partial Y}\right)_{z=0} = \left.\nu\,\frac{d^2U}{dz^2}\right)_{z=0}$$

$$\left.\nu\,\frac{d^2U}{dz^2}\right)_{z=0} : -0.152\left(m/s^2\right)\frac{(0.5\times10^{-3})^2}{(1.85\times10^{-3})^2} = -0.0109\left(m/s^2\right) \qquad \left(\text{for } u^* = 0.5\,m/s\right)$$

$$-0.152\left(m/s^2\right)\frac{(0.5\times10^{-3})^2}{(10^{-5})^2} = -0.76\times10^4\left(m/s^2\right) \qquad \left(\text{for } u^* = 0.4\,m/s\right)$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y}\cos\theta = -0{,}152 \times \cos 30^\circ = -0{,}132\left(m/s^2\right)$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial X} = -\frac{1}{\rho}\frac{\partial P}{\partial Y}\sin\theta = -0{,}152 \times \sin 30^\circ = -0.076\left(m/s^2\right)$$

(for $u^* = 0{,}45\ m/s$)

$$\left(-\frac{1}{\rho}\frac{\partial P}{\partial y}\right) = -0.0109 \times \cos 30^\circ = 9{,}4 \times 10^{-3}\left(m/s^2\right)$$

$$\left(-\frac{1}{\rho}\frac{\partial P}{\partial x}\right) = -0{,}0109 \times \sin 30^\circ = 5.5 \times 10^{-3}\left(m/s^2\right)$$

(for $u^* = 0{,}5\ m/s$)

Freestream at $z = 200\ m$

total velocity estimated with the Power Law

$$\left(\frac{z_{10}}{z_{200}}\right)^{\frac{1}{7}} = \frac{U_{10}}{U_{200}} \quad ; \quad \left(\frac{10}{200}\right)^{\frac{1}{7}} = \frac{15\ m/s}{U_{200}}$$

$$U_{200} = 22.8\ m/s$$

$$u_\infty = U_{200} \sin\theta = 22.8\ m/s \sin 30^\circ = 11.4\ m/s \ (-x \text{ dir})$$

$$v_\infty = U_{200} \cos\theta = 22.8\ m/s \cos 30^\circ = 19.7\ m/s \ (-y \text{ dir})$$

Freestream:

$$\frac{\partial u_\infty}{\partial t} = f v_\infty - \frac{1}{\rho}\frac{\partial P}{\partial x} \quad ; \quad \frac{\partial v_\infty}{\partial t} = f u_\infty - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$f = 2\Omega \sin\phi \; ; \quad \Omega = 7.2921\times 10^{-5}\,(s^{-1}) \;,\; \phi = 24^\circ$$

$$= 5\times 10^{-5}\,(s^{-1})$$

$\left(-\frac{1}{\rho}\frac{\partial P}{\partial x}\right)$: $f v_\infty = -5\times 10^{-5} \times 19.7 \sim -1\times 10^{-3}\ m/s^2$ (1Pa/km)

$\left(-\frac{1}{\rho}\frac{\partial P}{\partial y}\right)$: $f u_\infty = -5\times 10^{-5} \times 11.4 \sim -6\times 10^{-4}\ m/s^2$

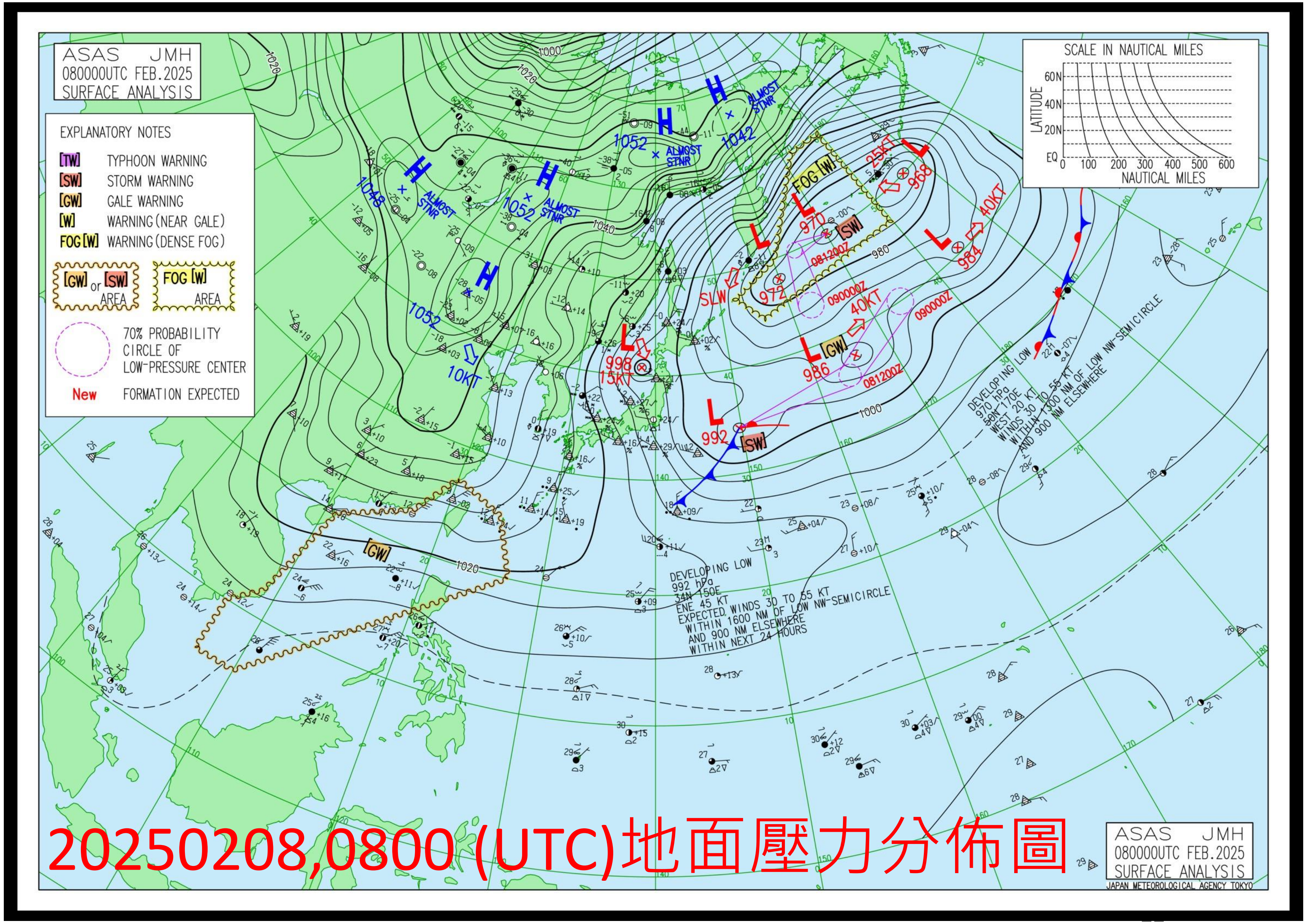
$$\left(f v_\infty\right) << \left(-\frac{1}{\rho}\frac{\partial P}{\partial x}\right) \quad ; \quad \left(f u_\infty\right) << \left(-\frac{1}{\rho}\frac{\partial P}{\partial y}\right)$$

Coriolis force — Pressure gradient ; Coriolis force — Pressure gradient

Evaluation of the pressure gradient force

High pressure system's pressure gradient is estimated in the order of 40 mb over 1000km.
For 1 mb=100 Pa,
it is about 4 Pa/km
or $4x10^{-3}$ (m/s^2).

This magnitude serves as a reference. The acceleration due to the terrain effect of Taiwan island should be remarked as an additional.



Concluding remarks

This exercise confirms the feasibility using the fully-roughen log velocity profile to obtain the aerodynamic roughness k_s.

k_s is much more sensitive to the uncertainty of friction velocity than z_0 reduced from the Charnock's empirical relation.

If u* between 0.45 and 0.5 m at U_{10}=15 m/s, k_s would fall in a range between 1 and 1.5 mm which is deemed physically sound.

For k_s between 1 mm and 1.5 mm, k_s is about 3 to 5 times z_0.

On the other hand, one may compare k_s and z_0 using the two empirical log velocity profiles below.

(a) $U_{10}/u^*=2.5 \ln(10/k_s)+8.5$ for aerodynamic boundary layers
(b) $U_{10}/u^*=2.5 \ln((10-d)/z_0)$ for neutrally stable environmental boundary layer

In (a) and (b), the reference elevation height is 10 m. In (b), d denotes the zero displacement, the elevation where the wind speed is zero empirically determined.

(a)=(b): $k_s/z_0 \approx 30\,((10-d)/10)$

It is concluded that k_s is greater than z_0, but no more than 30 times. The ratio depends on d, the state of ocean surface.

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-永傳公司提供福海離岸海氣象觀測塔進行數據資料收集

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