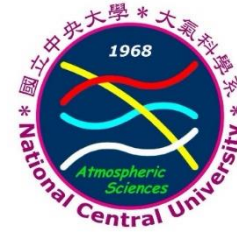


Implications of Western Pacific Tropical Cyclones' RSD's to the Bulk Microphysics Schemes and Quantitative Precipitation Estimation Using Machine Learning Models



Jayalakshmi Janapati, Balaji Kumar Seela, and Pay-Liam Lin*

**Planetary Boundary Layer and Air pollution Laboratory (PBLAP)
Department of Atmospheric Sciences, National Central University, Taiwan**

Introduction

- The microphysics parameterization schemes used in NWP models are
 - Bulk microphysics schemes
 - One moment
 - Two moment
 - Three moment
 - Spectral bin microphysics schemes
 - Lagrangian particle based schemes
- For operational purposes one moment bulk microphysics schemes are used because of efficiency and low computational cost.

Introduction

- These schemes assume the particle size distribution as some statistical distribution like exponential or gamma
- In one moment schemes two gamma parameters μ and N_0 are assumed as constants and the other parameter Λ can be diagnosed from the prognostic variable mass mixing ratio Q .
- But the assumption is not true!

Introduction

- How to improve one moment bulk microphysics schemes:
- Prescribe N_o from the look-up table
 - Calculate the second parameter Λ from a relationship ($\Lambda-N_o$) obtained from the observed RSD
 - The third parameter μ from the mass mixing ratio and the other two gamma parameters ($Q, \Lambda, N_o \rightarrow \mu$)

Data & methods

Data sets used (2005-2019)

- JTWC tropical cyclones track information
- JWD data from north Taiwan (NCU)

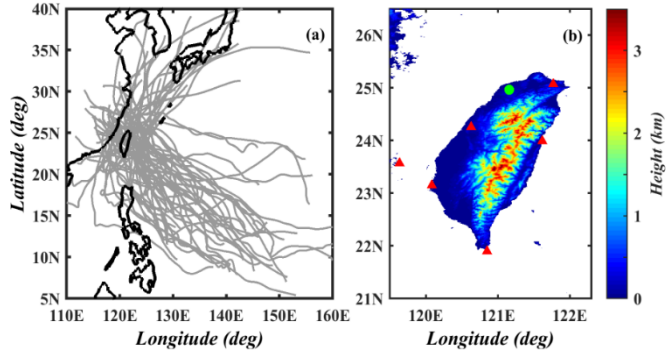


Fig. (a) WP TCs' tracks **(b)** location of JWD (green filled circle) in north Taiwan.

The rain drop concentration $N(D)$ ($m^{-3} mm^{-1}$) from the JWD is given as:

$$N(D)(m^{-3}mm^{-1}) = \sum_{i=1}^{20} \frac{n_i}{A \times \Delta t \times V(D_i) \times \Delta D_i}$$

where , n_i is the number of drops reckoned in the size bin i , A (m^2) and Δt are the sampling area and time, D_i (mm) is the drop diameter for the size bin i , ΔD_i is the corresponding diameter interval (mm), V_j (m/s) is terminal velocity of drops of diameter D_i

R (rainfall rate, $mm h^{-1}$), Z (radar reflectivity, dBZ), N_t (total number concentration, m^{-3}), and LWC (liquid water content, gm^{-3}) are estimated:

$$R (mm h^{-1}) = 6\pi \times 10^{-4} \sum_{i=1}^{20} V(D)N(D) D^3 \Delta D$$

$$Z (dBZ) = 10 \times \log_{10} \left(\sum_{i=1}^{20} N(D)D^6\Delta D \right)$$

$$N_t (m^{-3}) = \sum_{i=1}^{20} N(D) \Delta D$$

$$LWC (g m^{-3}) = \frac{\pi}{6} \rho_w \sum_{i=1}^{20} N(D) D^3 \Delta D$$

$$D_m (mm) = M4/M3 \quad M_n = \int_{D_{min}}^{D_{max}} D^n N(D)dD$$

TC rainfall @ disdrometer site:

Distance between TC center and JWD site ≤ 500 km

Gamma distribution and moments method:

The Gamma distribution function:

$$N(D) = N_o D^\mu \exp(-\Lambda D)$$

The nth moment of the gamma distribution:

$$M_n = \int_0^\infty D^n n(D) dD = N_o \Lambda^{-(\mu+n+1)} \Gamma(\mu + n + 1)$$

$M_2 M_3 M_4$, $M_2 M_4 M_6$, $M_3 M_4 M_6$, consecutive moments $M_n M_{n+1} M_{n+2}$

The gamma distribution parameters in terms of 2nd, 3rd, and 4th moments (M_2 , M_3 , and M_4) are expressed as (Smith and Kliche, 2005):

$$\mu = \frac{Q(m+1) - (m+2)}{1-Q} \quad \lambda = \frac{M_2(m+\mu+1)}{M_3} \quad N_0 = \frac{M_m \Lambda^{(m+\mu+1)}}{\Gamma(m+\mu+1)}$$

Here, $Q = \frac{M_2 M_4}{M_3^2}$. With $m=2$, the above shape, slope and intercept parameters corresponds to 2nd, 3rd and 4th moments

The gamma distribution parameters in terms of 2nd, 4th, and 6th moments (M_2 , M_4 , M_6) are expressed as (Ulbrich and Atlas, 1998):

$$\mu = \frac{(7 - 11\eta) - \sqrt{\eta^2 - 14\eta + 1}}{2(\eta - 1)} \quad \lambda = \sqrt{\frac{M_2 \Gamma(\mu + 5)}{M_4 \Gamma(\mu + 3)}} \quad N_0 = \frac{M_4 \Lambda^{(\mu+5)}}{\Gamma(\mu + 5)} \quad \text{Here, } \eta = \frac{M_4^2}{M_2 M_6}$$

Gamma distribution and moments method:

The gamma distribution parameters in terms of 3rd, 4th, and 6th moments (M_3, M_4, M_6) are expressed as (Ulbrich, 1983):

$$\mu = \frac{(11G - 8) + \sqrt{G(G + 8)}}{2(1 - G)} \quad \lambda = \frac{M_3(\mu + 4)}{M_4} \quad N_0 = \frac{M_3 \Lambda^{(\mu+4)}}{\Gamma(\mu + 4)} \quad \text{Here, } G = \frac{M_4^3}{M_3^2 M_6}$$

The gamma distribution parameters can be expressed by any three consecutive moments (i.e., M_n, M_{n+1}, M_{n+2}) can be expressed as (Smith, 2003; Smith et al., 2009)

$$\mu = \frac{B(n + 1) - (n + 2)}{(1 - D)} \quad \Lambda = \frac{M_n(n + \mu + 1)}{M_{n+1}} \quad N_0 = \frac{M_n \Lambda^{(n+\mu+1)}}{\Gamma(n + \mu + 1)} \quad \text{Where, } B = \frac{M_n M_{n+2}}{M_{n+1}^2}$$

Results & Discussion

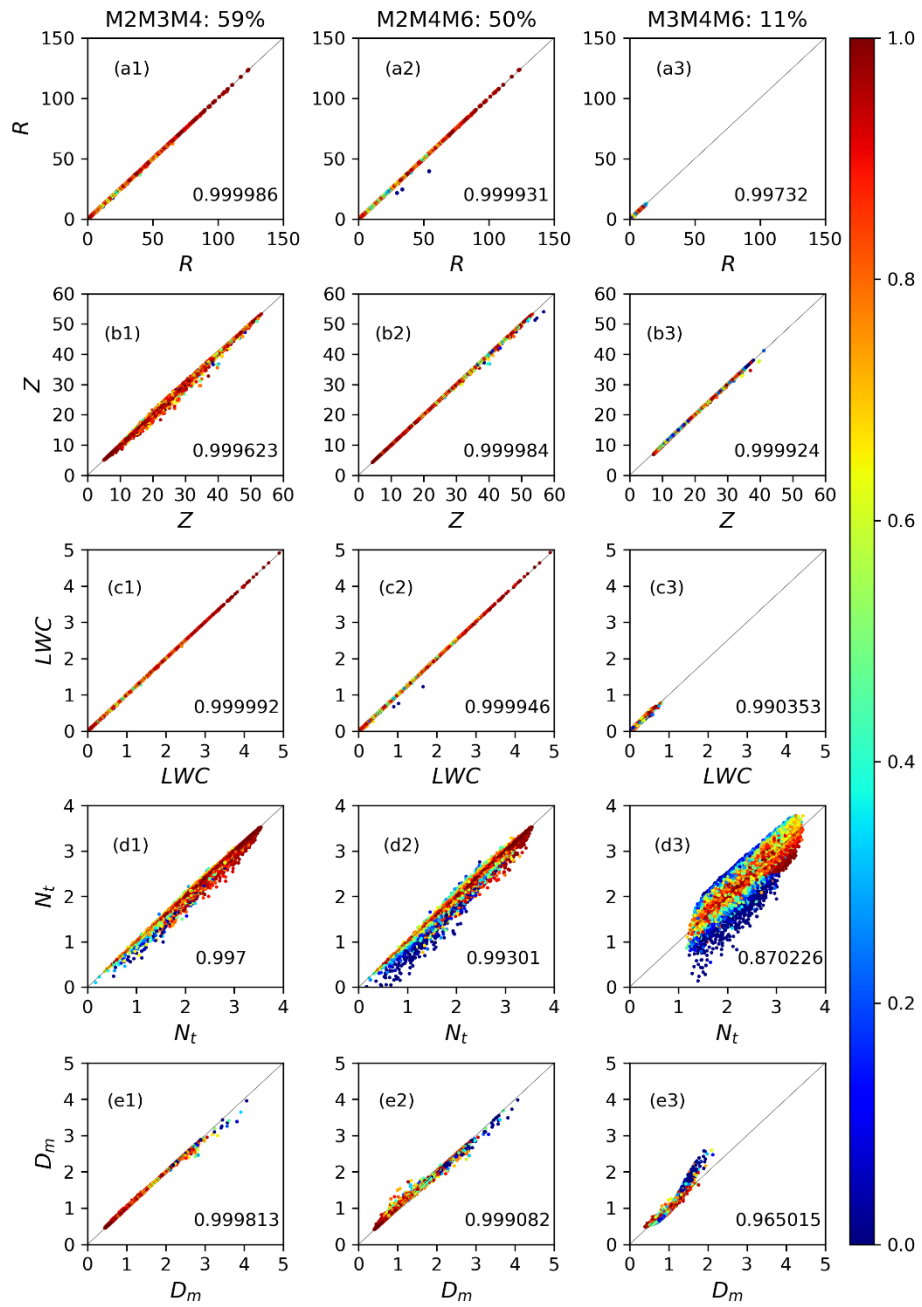


Figure. Comparison of RSD parameters (R in mm h^{-1} : first row, Z in dBZ : second row, LWC in g m^{-3} : third row, N_t in m^{-3} : fourth row, and D_m in mm : fifth row) estimated from the observation (X-axis) and gamma distribution (GD) fits (Y-axis) using second, third, and fourth moments (M₂M₃M₄: first column), second, fourth, and sixth moments (M₂M₄M₆: second column), third, fourth, and sixth moments (M₃M₄M₆: third column).

Results & Discussion

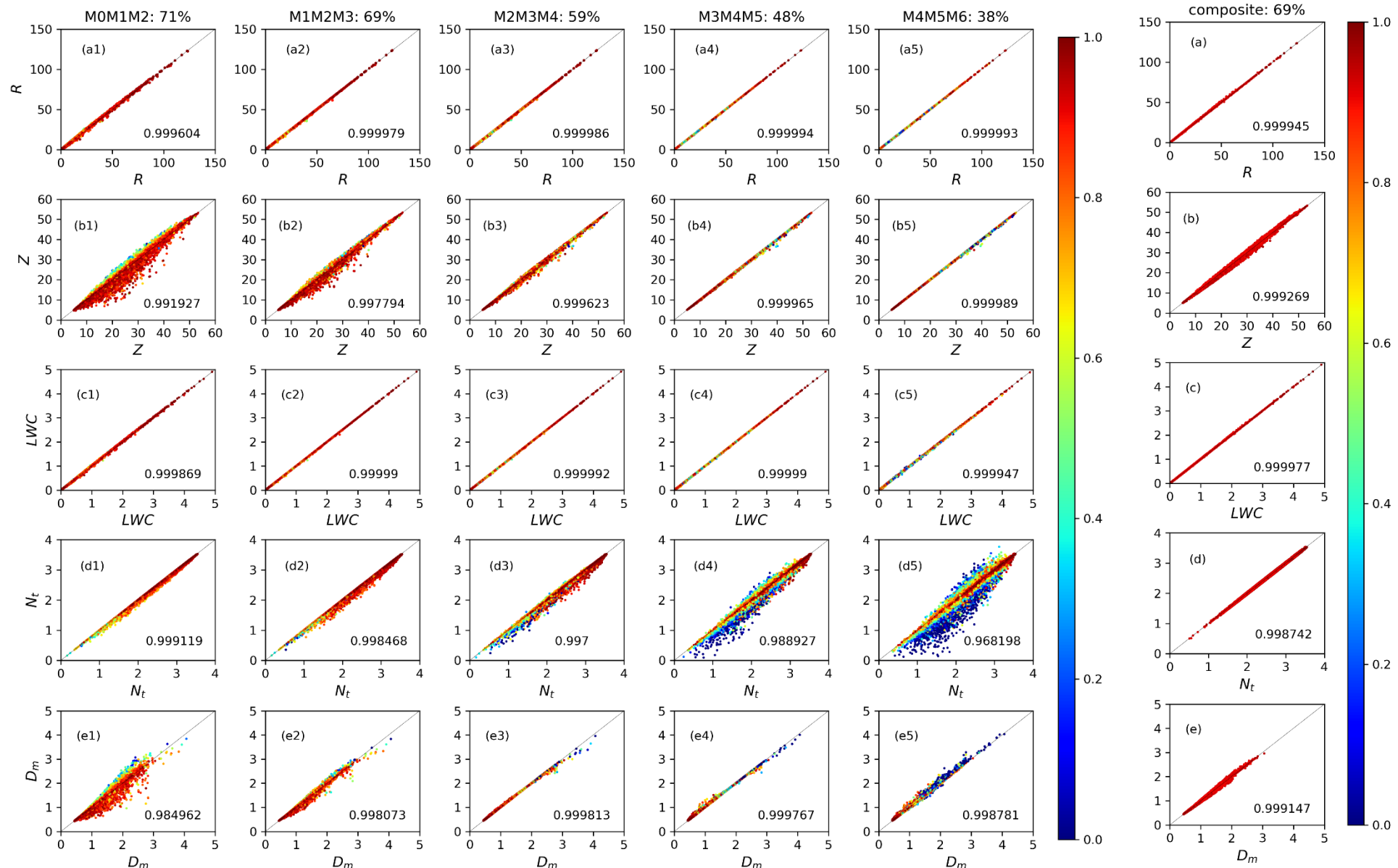


Figure. Comparison of RSD parameters (R in mm h⁻¹: first row, Z in dBZ: second row, LWC in g m⁻³: third row, N_t in m⁻³: fourth row, and D_m in mm: fifth row) estimated from the observation (X-axis) and gamma distribution (GD) fits (Y-axis) using three consecutive moments (M_{012} : first column, M_{123} : second column, M_{234} : third column, M_{345} : fourth column, M_{456} : fifth column) and hybrid moments (sixth column).

Relationships among gamma parameters (μ , λ and N_o)

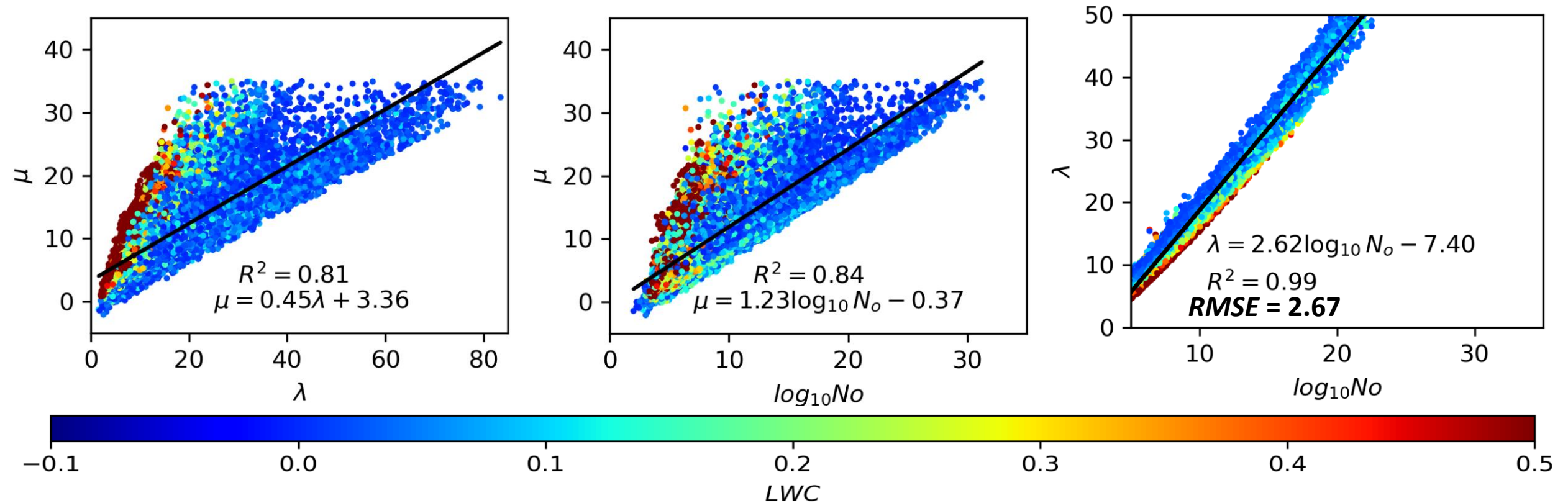


Figure. Scatter plots of shape–slope parameter (μ – λ), shape–intercept parameter (μ – $\log_{10}N_o$), and slope–intercept parameter (λ – $\log_{10}N_o$), and with liquid water content (LWC).

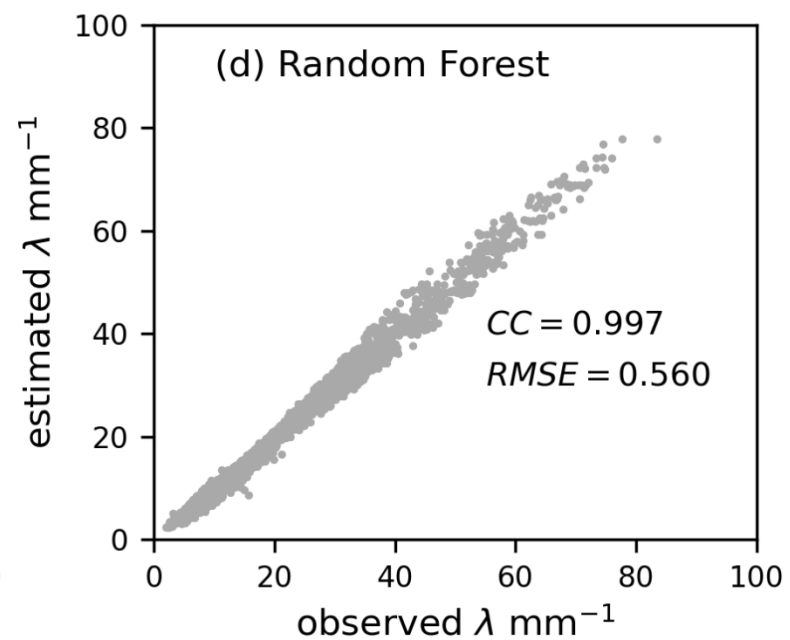
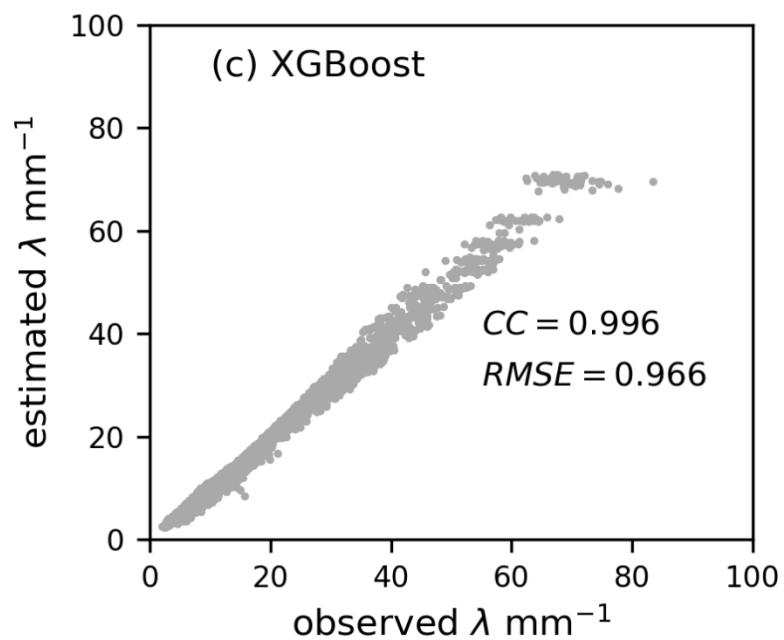
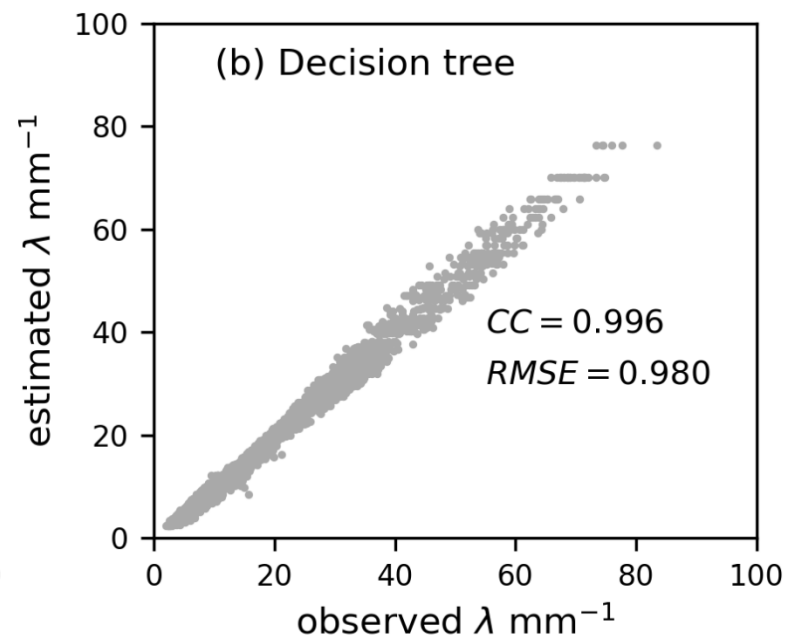
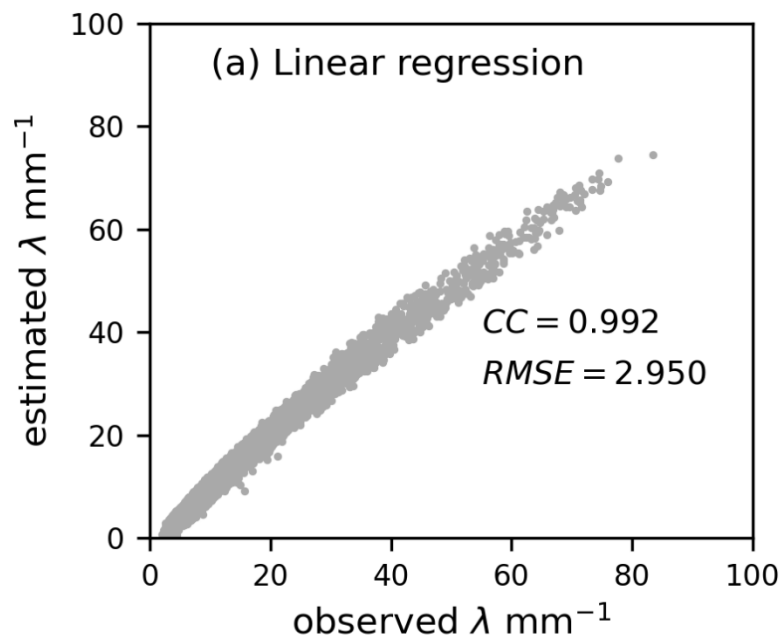
Machine learning models:

1. Decision Tree
2. XGBoost
3. Random Forest

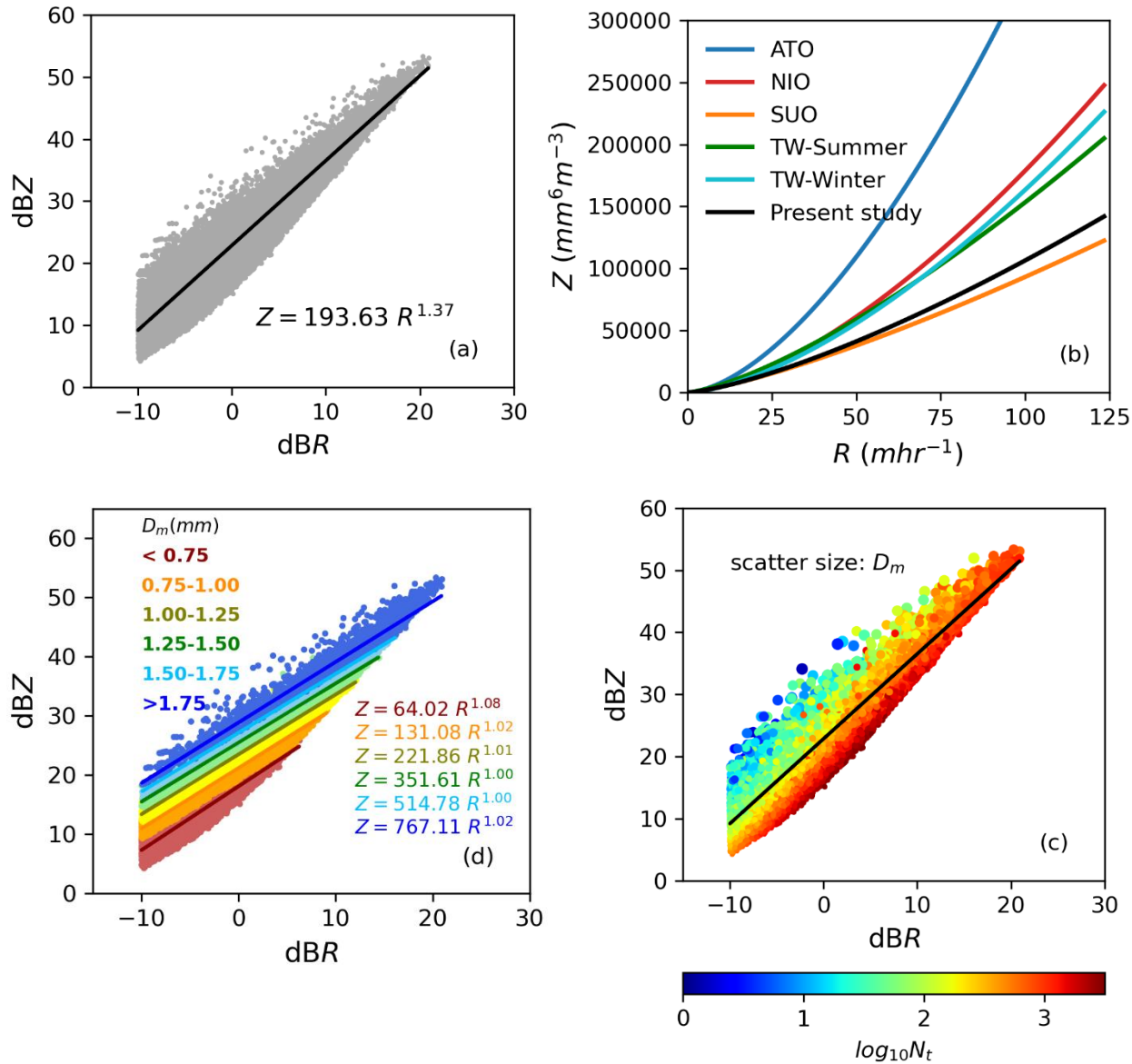
Training data – 70%

Test data – 30%

Results & Discussion



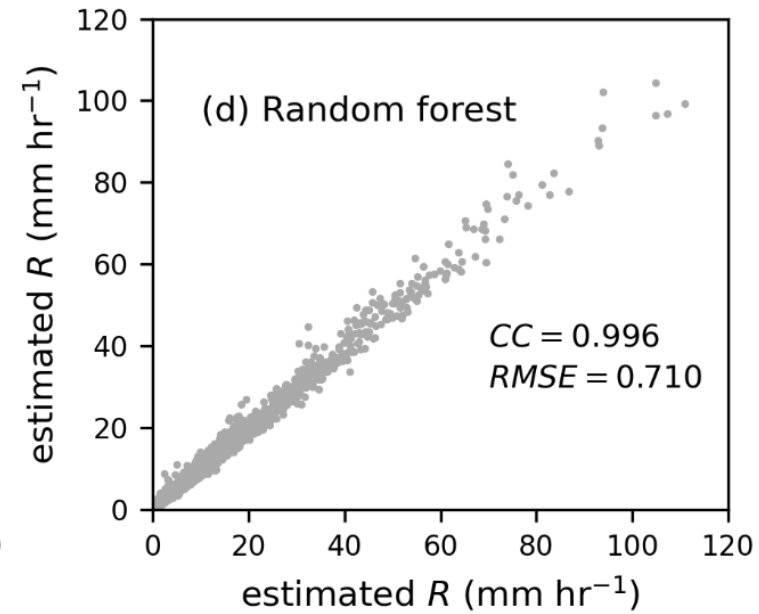
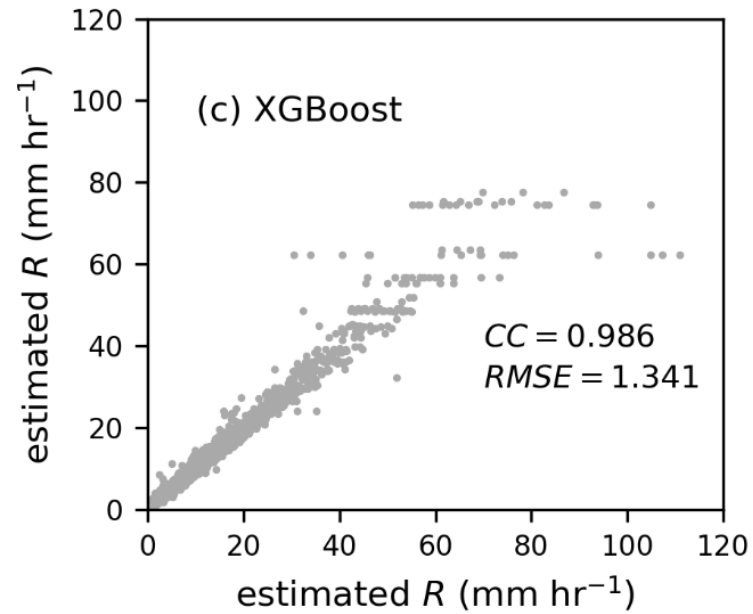
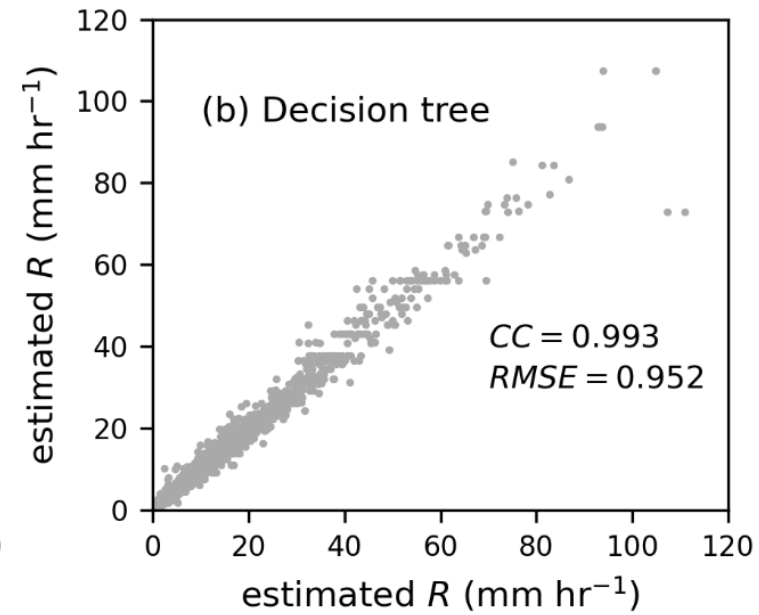
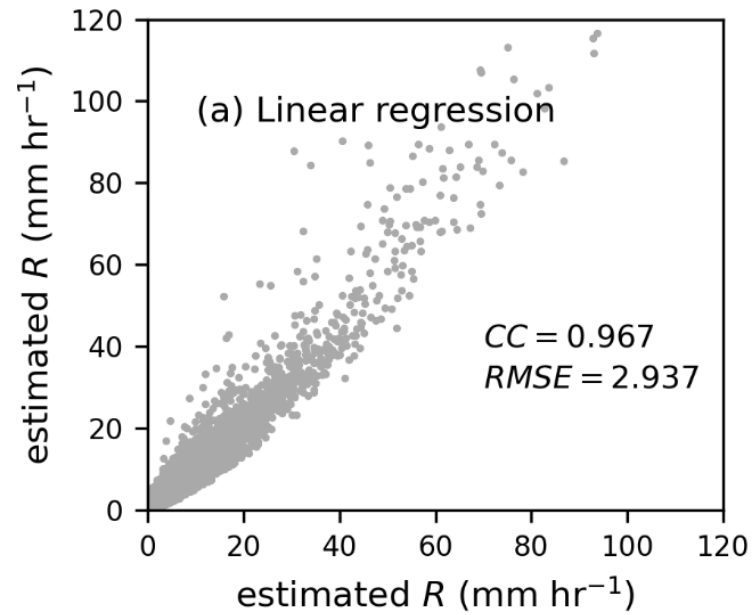
Radar reflectivity and rainfall rate ($Z-R$) relations



Oceanic region	Z-R relations	Reference
Atlantic Ocean (ATO)	$Z = 186.00 R^{1.63}$	Hopper et al., (2020)
North Indian ocean (NIO)	$Z = 142.04 R^{1.55}$	Radhakrishna and Narayana Rao, (2010)
Southern Ocean (SUO)	$Z = 234.00 R^{1.3}$	Deo and Walsh, (2016)
Taiwan-Summer	$Z = 266.42 R^{1.38}$	Seela et al., (2018)
Taiwan-Winter	$Z = 129.76 R^{1.55}$	Seela et al., (2018)

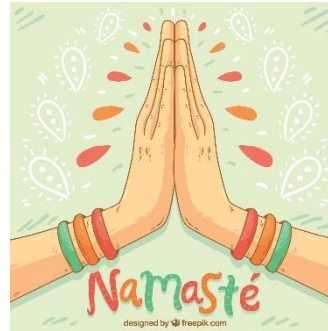
Figure. Radar reflectivity and rainfall rate relations.

Results & Discussion



Conclusions

- The radar reflectivity and rainfall rate relations of Western Pacific tropical cyclones (WP TCs) are distinctly different from that of the other oceanic TCs.
- The rain fall rates estimated with Machine learning approaches (inputs: Z & D_m) showed superior performance over the linear regression Z-R relation.
- The RSD parameters estimated with hybrid/composite moments over the fixed three moments method are well agreed with observation.
- Among slope-shape, shape-slope, and slope-intercept parameter relations, the he three kind relationships among slope, shape and intercept parameters parameters are close to one-to-one line for Slope–intercept parameter ($\lambda - \log_{10}N_0$),
- Among shape–slope parameter ($\mu - \lambda$), slope–intercept parameter ($\lambda - \log_{10}N_0$), shape–intercept parameter ($\mu - \log_{10}N_0$) relations, the less spread in data points from one-to-one line is observed for $\lambda - \log_{10}N_0$.
- The slope parameters estimated using machine learning approaches (inputs: $\log_{10}N_0$ and LWC) demonstrated better results over the linear $\lambda - \log_{10}N_0$ relations.
- Present study demonstrates that, better estimates of rainfall rates and slope parameter can be achieved through random forest approach over linear regression methods.



Thank you for listening !!

n^{th} Moment (M_n):

$$M_n = \int_{D_{\min}}^{D_{\max}} D^n N(D) dD$$

Moment	Microphysics Variable
0	Particle/drop concentration (m^{-3}): $N = M_0 = \int_0^{\infty} n(D) dD$
3	Mass mixing ratio ($kg \cdot kg^{-1}$): $Q = M_3 = \frac{\pi \rho_w}{\rho} \int_0^{\infty} D^3 n(D) dD$
6	Radar Reflectivity ($mm^6 m^{-3}$): $Z = M_6 = \int_0^{\infty} D^6 n(D) dD$

Microphysics parameterization schemes

- Bulk microphysics schemes: assume PSD/RSD/DSD to be certain functions
 - One moment (1)
 - Two moment (2)
 - Three moment (3)
- Spectral bin microphysics schemes: calculate PSDs/DSDs/RSDs by solving explicit microphysical equations
- Lagrangian particle-based schemes

One moment Microphysics Parameterization Schemes:

$$\frac{\partial Q_R}{\partial t} = \underbrace{-V \cdot \nabla Q_R}_{\text{Advection term}} + \underbrace{\nabla \cdot K_m \nabla Q_R}_{\text{Diffusion term}} + \underbrace{P_R}_{\text{Production term}} + \underbrace{\frac{1}{\rho} \frac{\partial (\rho U_R Q_R)}{\partial z}}_{\text{Fallout term}}$$

$$P_R = P_{RAUT} + P_{RACW} + \dots$$

$$P_{RACW} = \frac{\pi E_{RW} n_{OR} a Q_{CW} \Gamma(3 + b)}{4 \lambda_R^{3+b}} \left(\frac{\rho_0}{\rho} \right)^{1/2}$$

$$U_R = \frac{a \Gamma(4 + b)}{6 \lambda_R^b} \left(\frac{\rho_0}{\rho} \right)^{1/2}$$

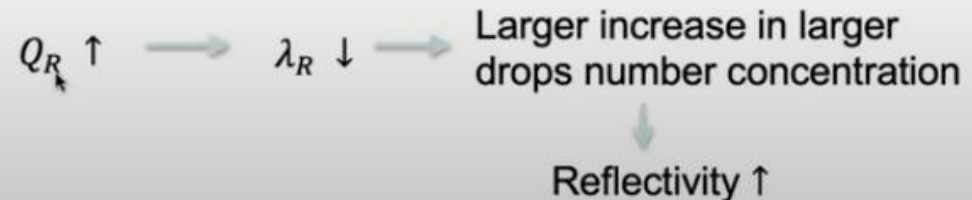
$$n_R(D) = n_{OR} e^{-\lambda_R D}$$

Lin et al. 1983;

$$n_{OR}(\text{intercept}) = 8 \times 10^{-2} \text{ cm}^{-4}$$

*GFDL microphysics
scheme in HALFS*

$$\lambda_R(\text{slope}) = \left(\frac{\pi \rho_W n_{OR}}{\rho Q_R} \right)^{0.25}$$



Improve bulk microphysics parameterization schemes

- One-moment bulk microphysics parameterization Schemes
 - Prescribe one parameter through a look-up table;
 - Calculate the second parameter from a relationship identified from the observations;
 - Diagnose the third parameter from the mixing ratio;
- Two-moment bulk microphysics parameterization Schemes
 - A gamma distribution can be fully determined from a relationship identified from the observations and two prognostic variables (mixing ratio and number concentration)