Definition and Parameterization of Unresolved Convection in Cloud-Resolving NWP Models

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Outline of this presentation

1. Reynolds average vs spatial filter for resolved and unresolved scale separation

2. Multi-fluid vs spatial filter

3. Parameterization of unresolved convection: the multi-fluid vs spatial filter framework

4. A unified PBL and convection parameterization for CRMs

5. Preliminary numerical examples in the Unified Forecast System (UFS)

6. Summary

Full prognostic equation

All the prognostic equations of atmospheric motion can be summarized into the following form:

$$\frac{\partial}{\partial t}\varphi + \nabla \cdot \boldsymbol{u}\varphi + \frac{1}{\rho}\frac{\partial}{\partial z}\rho w\varphi = F$$

for an arbitrary prognostic variable φ.

There are four conceptual steps to solve the above equation:

- A numerical discretization to define resolved and unresolved scales
- Discretized governing equations for the resolved scale including the convergence of sub-grid flux associated with unsolved motion
- Parameterization representation of unresolved sub-grid flux
- Numerical solution using a computer

Reynolds scale separation rules based on an average over a grid box (Δ^3)

$$\bar{\varphi}(X,Y,Z,t) = \lim_{\Delta\to\infty} \frac{1}{\Delta^3} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \varphi(X-x, Y-y, Z-z, t)\, dxdydz$$

The meaning of $\Delta\to\infty$ is that the unresolved scales are required to be much smaller than the smallest resolved scale. This average operation has the following properties:

1. The average of a sum is the sum of the averages (the distributive property).

2. The average of a derivative is the derivative of the average (the commutative property).

3. $\overline{\varphi'} \equiv \overline{\varphi - \bar{\varphi}} = 0$

Generalization of scale separation based on a spatially filtered average with a characteristic scale (Δ)

$$\bar{\varphi}(X,Y,Z,t) = \frac{1}{L_x L_y L_z}\int_{-\frac{L_x}{2}}^{\frac{L_x}{2}}\int_{-\frac{L_y}{2}}^{\frac{L_y}{2}}\int_{-\frac{L_z}{2}}^{\frac{L_z}{2}} G(\Delta; X-x, Y-y, Z-z, t)\,\varphi(x,y,z,t)dxdydz$$

1. The average of a sum is the sum of the averages (the distributive property).

2. The average of a derivative is the derivative of the average (the commutative property).

3. $\overline{\varphi'} \equiv \overline{\varphi - \bar{\varphi}} \neq 0$

Resolved prognostic equation

$$\frac{\partial}{\partial t}\bar{\varphi} + \nabla \cdot \overline{\boldsymbol{u}\varphi} + \frac{1}{\rho}\frac{\partial}{\partial z}(\rho\overline{w\varphi}) = Q,$$

where Q is the so-called apparent source defined in terms of the sub-scale variables and resolved forcing as

$$Q = -\nabla \cdot \overline{\boldsymbol{u}'\varphi'} - \frac{1}{\rho}\frac{\partial}{\partial z}(\rho\overline{w'\varphi'}) + \bar{F}\ ,$$

when the Renolds average is used, or

$$Q = -\nabla \cdot (\overline{\boldsymbol{u}\varphi} - \bar{\boldsymbol{u}}\bar{\varphi}) - \frac{1}{\rho}\frac{\partial}{\partial z}[\rho(\overline{w\varphi} - \bar{w}\bar{\varphi})] + F,$$

when the general filter is applied.

Multi-fluid approach: Segmentally-constant approximation

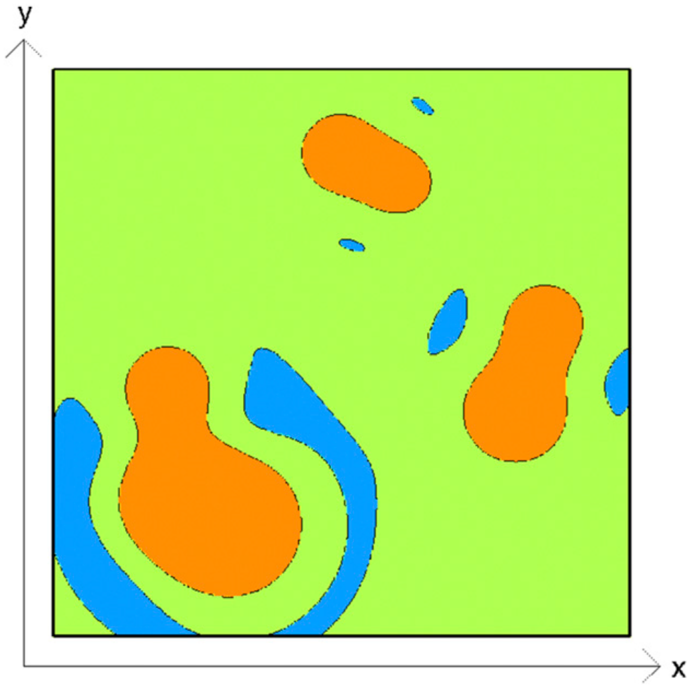


FIG. 1. Schematic horizontal section showing a decomposition of the fluid into multiple components, e.g., updrafts (orange), downdrafts (blue), and the environment (green). In each component, one of the I_i values is equal to 1, and the others are equal to 0.

From Thuburn et al. (2018)

SCA approximates a full system by a set of constant segments designated by areas S_j and corresponding boundaries ∂S_j with the indices $j = 1, 2, \ldots n$ at each vertical level. These areas may be enclosed (as updrafts and downdrafts) or open (as for the environment), but the whole grid-box domain is subdivided into those segments so that the sum of the areas for all segments recover the total grid-box area, *i.e.*, $S = \sum_{j=1}^{n} S_j$. The basic concept is already schematically depicted by Figs. 1–3. Alternatively, each segment may be considered in analogous manner as cloud types as shown by Fig. 2 of Yano et al. (2005a), and Fig. 1 of Yano (2012a).

All the physical variables under SCA may be defined by

$$\varphi = \sum_{j=1}^{n} \mathcal{I}_j(x, y, z)\varphi_j,$$

where $\mathcal{I}_j(x, y, z)$ is an indicator for the jth segment, which is defined by

$$\mathcal{I}_j(x, y, z) = \begin{cases} 1, & \text{if } (x, y, z) \in S_j, \\ 0, & \text{if } (x, y, z) \notin S_j. \end{cases}$$

From Yano (2014, *Dynamics of Atmospheres and Oceans*)

Multi-fluid system: Segmentally-constant approximation

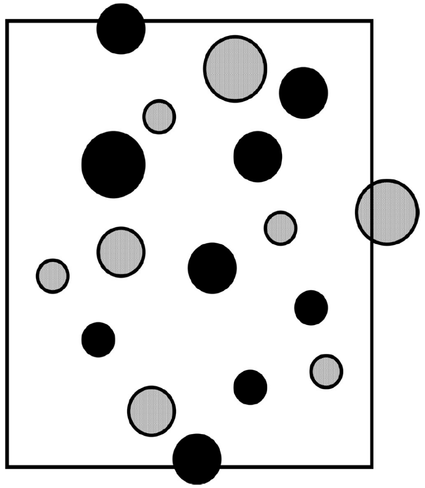


Fig. 2. A generalization of Riehl and Malkus' hot-tower hypothesis in Fig. 1 into two convective-scale components: dark circles and gray circles representing updrafts and downdrafts, respectively. This corresponds to a special case of segmentally-constant approximation (SCA) over subgrid-scale processes.

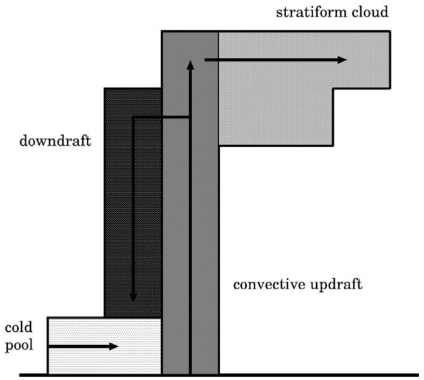


Fig. 3. A side view for a further generalization of SCA. Unlike the case of Fig. 2, the subgrid-scale components are no longer exclusively interacting with the environment, but with various other components: convective updraft, downdraft, cold pool, stratiform cloud.

From Yano (2014)

Separate equations of motion for the stable environment ($i = 0$), the convective plumes ($i = 1$) and downdrafts ($i = 2$):

$$\frac{\partial \sigma_i \rho_i}{\partial t} + \nabla \cdot (\sigma_i \rho_i \mathbf{u}_i) = \sum_{j \neq i} (\sigma_j \rho_j S_{ji} - \sigma_i \rho_i S_{ij})$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i + 2\Omega \times \mathbf{u}_i + c_p \theta_i \nabla \pi_i = \mathbf{g} + \sum_{j \neq i} \left(\frac{\sigma_j \rho_j}{\sigma_i \rho_i} S_{ji} (\mathbf{u}_{ji}^T - \mathbf{u}_i) - S_{ij} (\mathbf{u}_{ij}^T - \mathbf{u}_i) - \mathbf{D}_{ij} \right)$$

$$\frac{\partial \theta_i}{\partial t} + \mathbf{u}_i \cdot \nabla \theta_i = \sum_{j \neq i} \left(\frac{\sigma_j \rho_j}{\sigma_i \rho_i} S_{ji} (\theta_{ji}^T - \theta_i) - S_{ij} (\theta_{ji}^T - \theta_i) \right)$$

σ_i is the volume fraction of fluid i and $\sum_i \sigma_i = 1$

$\sigma_i \rho_i S_{ij}$ is the mass of fluid transferred from i to fluid j

θ_{ij}^T is the potential temperature of the fluid transferred from i to fluid j

$\mathbf{u}_{ij}^T$ is the velocity of the fluid transferred from i to fluid j

π_i is the Exner pressure in fluid i, $p_0 \pi_i^{\frac{1-\kappa}{\kappa}} = R \rho_i \theta_i$

$\mathbf{D}_{ij}$ is drag on fluid i from fluid j

From Weller et al. (2020)

Two potential issues in the representation of subgrid convection in the UFS

1. A theoretical ambiguity in making conventional subgrid convection parameterization schemes scale aware for applications at the so-called gray-zone resolutions

2. The *ad hoc* separation of vertical scales of parameterization subgrid convection into, e.g., deep/shallow and PBL convection

The approach in the UFS for implementing the scale-aware convection scheme (Arakawa et al., 2011)

In the aforementioned multi-fluid system, we can show that

$$\overline{w'\psi'} = \sigma(1-\sigma)\Delta w \Delta\psi, \tag{1}$$

where

$$\overline{(\ )} = \sigma(\ )_c + (1-\sigma)(\tilde{\ }) \text{ and } \Delta(\ ) \equiv (\ )_c - (\tilde{\ })\,, \tag{2}$$

σ is the fractional area covered by the updraft, an overbar denotes a domain mean, the subscript *c* denotes a cloud value, and a tilde denotes an environmental value.

Define $(\overline{w'\psi'})_E$ as the flux required to maintain quasi-equilibrium. The closure assumption used to determine σ is

$$\sigma = \frac{(\overline{w'\psi'})_E}{\Delta w \Delta\psi + (\overline{w'\psi'})_E}\,. \tag{3}$$

The quantities on the right-hand side of (3) are expected to be independent of σ . Eq. (3) is guaranteed to give

$$0 \le \sigma \le 1\,. \tag{4}$$

By combining (3) and (1), we obtain

$$\overline{w'\psi'} = (1-\sigma)^2(\overline{w'\psi'})_E\,. \tag{5}$$

In the UFS, $(\overline{w'\psi'})_E$ is specified by a conventional parameterization formulation with a steady/diagnostic cloud model (aka the SAS scheme).

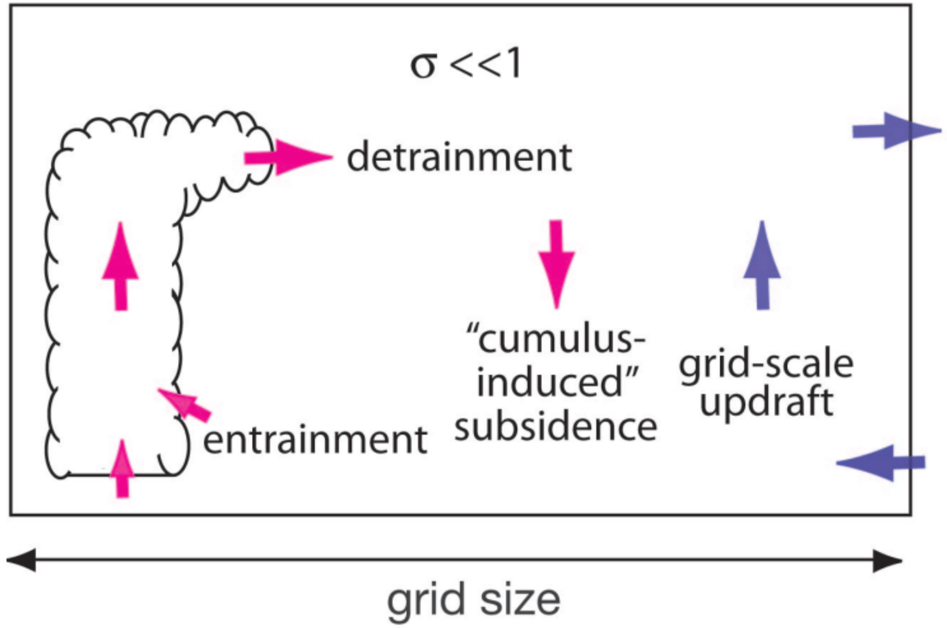
Adapted from a class note of Prof. Dave Randall

Ambiguity in implementing scale-aware convection schemes

- A theoretical explanation for the observed smallness of the fractional area covered by cumulus updrafts was provided by Bjerknes (1938, *QJRMS*, 64, 325-330).

- Convection takes place physically by rapid rising motion in the cloudy region, and slow sinking motion in the clear region.

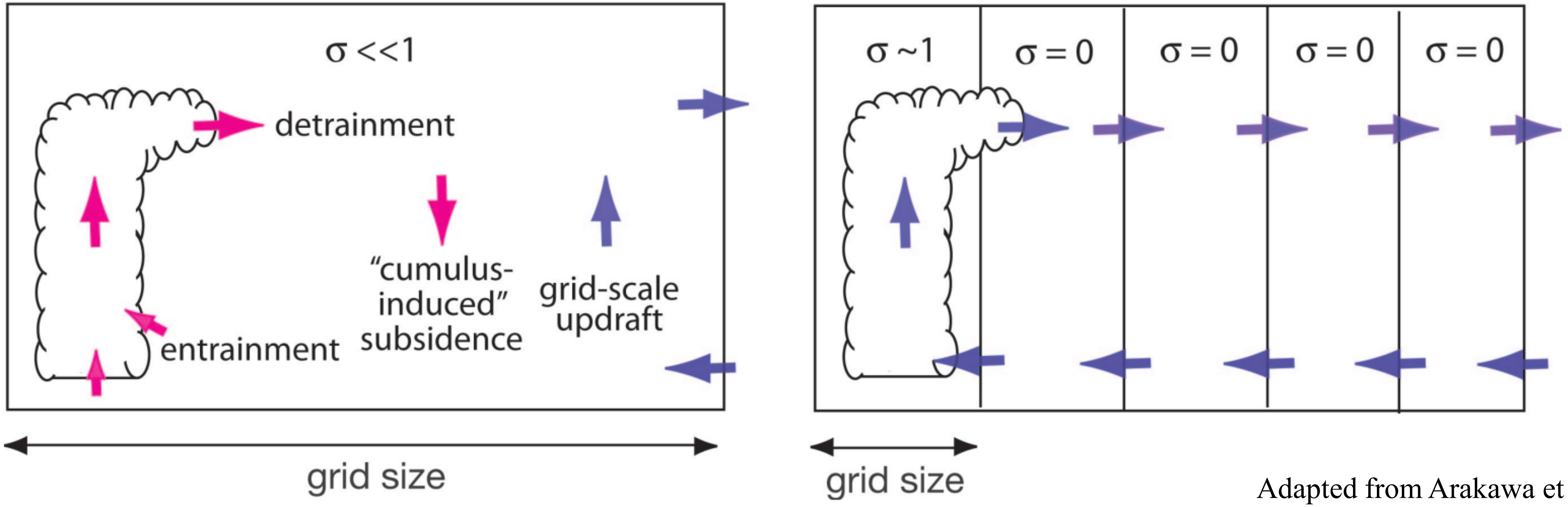
- In a grid box, mass conservation dictates that rapid rising motion and slow sinking motion can only be achieved physically, for a given value of $(w_c - \widetilde{w})$, by making the updraft narrow, and the downdraft broad, i.e., $\sigma \ll 1$.



Adapted from Arakawa et al. (2011, *ACP*)

Ambiguity in implementing scale-aware convection schemes

- The conventional convection parameterization using a steady cloud model for full adjustment to maintain quasi-equilibrium in the gray-zone resolution: (a) inconsistent with the assumption of $\sigma \ll 1$, and (b) coarse-graining statistics only relevant to deep convection.
- A good cloud model to determine $(\psi_x - \bar{\psi})$ and a reasonable closure to determine the magnitude of $(\overline{w'\psi'})_E$: What are practical metrics for being "good" and "reasonable"?



Adapted from Arakawa et al. (2011, *ACP*)

Multi-fluid system: Segmentally-constant approximation

Separate equations of motion for the stable environment ($i = 0$), the convective plumes ($i = 1$) and downdrafts ($i = 2$):

$$\frac{\partial \sigma_i \rho_i}{\partial t} + \nabla \cdot (\sigma_i \rho_i \mathbf{u}_i) = \sum_{j \neq i} (\sigma_j \rho_j S_{ji} - \sigma_i \rho_i S_{ij})$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i + 2\Omega \times \mathbf{u}_i + c_p \theta_i \nabla \pi_i = \mathbf{g} + \sum_{j \neq i} \left(\frac{\sigma_j \rho_j}{\sigma_i \rho_i} S_{ji} (\mathbf{u}^T_{ji} - \mathbf{u}_i) - S_{ij} (\mathbf{u}^T_{ij} - \mathbf{u}_i) - \mathbf{D}_{ij} \right)$$

$$\frac{\partial \theta_i}{\partial t} + \mathbf{u}_i \cdot \nabla \theta_i = \sum_{j \neq i} \left(\frac{\sigma_j \rho_j}{\sigma_i \rho_i} S_{ji} (\theta^T_{ji} - \theta_i) - S_{ij} (\theta^T_{ji} - \theta_i) \right)$$

σ_i is the volume fraction of fluid i and $\sum_i \sigma_i = 1$

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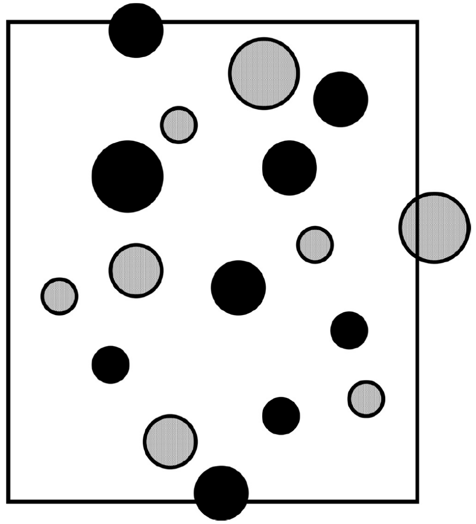
$\mathbf{D}_{ij}$ is drag on fluid i from fluid j

From Weller et al. (2020)

Mass flux and separation of convection and environment



$$\sigma_e = 1 - \sigma_c$$

with the latter defined by

$$\sigma_c = \sum_{j=1, j\neq e}^{n} \sigma_j.$$

Note that environmental mass flux is defined by

$$M_e = \rho\sigma_e w_e.$$

As we are going to see immediately below, under the entrainment–detrainment hypothesis, a quantity called the mass flux

$$M_j = \rho\sigma_j w_j \tag{5.7}$$

(Eqs. AS.2, YEC.17), is going to play a key role. This is the main reason that this parameterization formulation is coined "mass flux". This quantity characterizes the convective vertical transport of physical variables.

By substituting the definition of the entrainment and the detrainment rates (5.5), the mass continuity (4.9) reduces to

$$\frac{\partial}{\partial t}\sigma_j + \frac{1}{\rho}(D_j - E_j) + \overline{\nabla}\cdot\sigma_j \mathbf{u}_j. + \frac{1}{\rho}\frac{\partial}{\partial z}M_j = 0, \tag{5.8}$$

Finally, by taking a sum of Eq. (6.3) and (6.6) we obtain a prognostic equation for the grid-box mean:

$$\frac{\partial}{\partial t}\overline{\varphi} + \overline{\nabla}\cdot\overline{\mathbf{u}\varphi} + \frac{1}{\rho}\frac{\partial}{\partial z}\rho\overline{w\varphi} = \left(\frac{\partial\varphi}{\partial t}\right)_e + \sum_{j=1, j\neq e}^{n}\left(\frac{\partial\varphi}{\partial t}\right)_j \tag{6.8}$$

(cf., Eqs. AS.33, 34 and Eqs. AS.35, 36), where the right-hand side terms are defined by

$$\left(\frac{\partial\varphi}{\partial t}\right)_j = \sigma_j F_j - \frac{1}{\rho}\frac{\partial}{\partial z}M_j\varphi'_j - \overline{\nabla}\cdot\sigma_j\mathbf{u}_j\varphi'_j, \tag{6.9a}$$

$$\left(\frac{\partial\varphi}{\partial t}\right)_e = \sigma_e F_e - \frac{1}{\rho}\frac{\partial}{\partial z}M_e\varphi'_e - \overline{\nabla}\cdot\sigma_e\mathbf{u}_e\varphi'_e, \tag{6.9b}$$

where $\varphi'_j = \varphi_j - \overline{\varphi}$ is the deviation from the grid-box mean.

From Yano (2014, *Dynamics of Atmospheres and Oceans*)

Conventional formulation as the asymptotic limit of

$\sum_i \sigma_i = \sigma \ll 1$

The prognostic equation for jth subgrid-scale component under the limit of $\sigma_j \to 0$ are the same as already given by Eqs. (6.11) and (6.12), but φ_e replacing with $\overline{\varphi}$. Introduction of the steady-plume hypothesis (7.6), however, further simplifies the matter, and the prognostic equation reduces to a diagnostic form:

$$\frac{\partial}{\partial z} M_j \varphi_j = E_j \overline{\varphi} - D_j \varphi_j + \rho \sigma_j F_j. \tag{7.11}$$

By taking the limit $\sigma_j \to 0$ in Eq. (6.13), the mass continuity for the jth convective element becomes:

$$\frac{\partial}{\partial z} M_j = E_j - D_j \tag{7.12}$$

That is, in a conventional parameterization scheme with a steady/diagnostic cloud model in which the mass conservation is expressed by (7.12), $\sigma \ll 1$ is assumed.

From Yano (2014, *Dynamics of Atmospheres and Oceans*)

A pathway for avoiding the ambiguity:
A unified *scale-adaptive* concept
by Park (2014) that assumes the unresolved convective updraft/downdraft is relative to the grid-mean/filtered model state.

The unified vs the conventional formulation

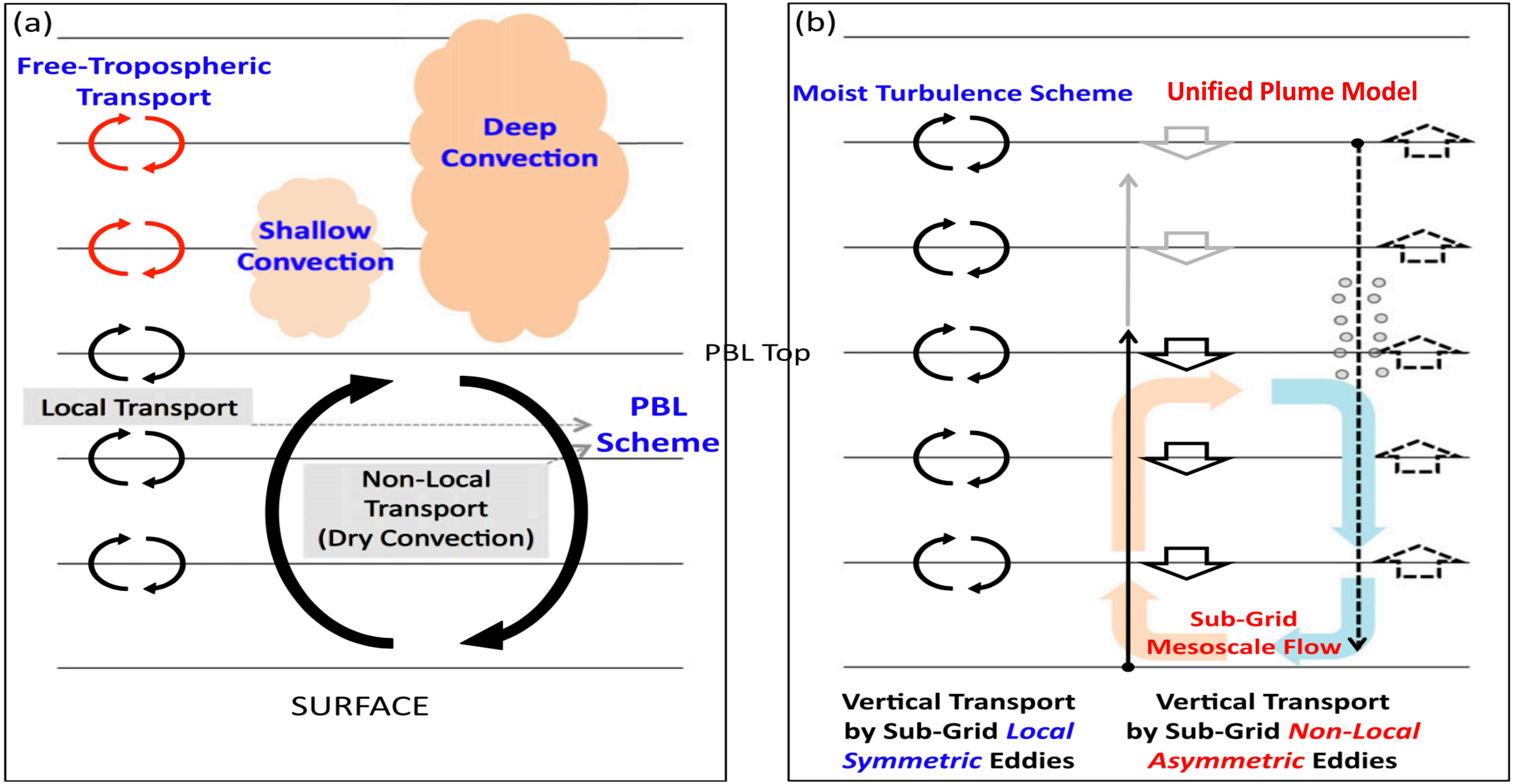
- Conventional formulation
  - The convection scheme simulates the actual convective motion on the subgrid scale.
  - The model grid mean vertical velocity is defined in terms of the multi-fluid average by

$$\overline{w} = (1-\sigma)\overline{w}_{env} + \sigma\overline{w}_{con}\ (\overline{w}_{con} \geq 0, \overline{w}_{env} \approx \overline{w}_{model}\ when\ \sigma \ll 1)$$

- Scale-adaptive formulation
  - The convection scheme simulates unresolved convective motion and treats it as the subfilter/unresolved solution of the model relative to the filtered/resolved solution.
  - The model grid mean vertical velocity is treated as the filtered solution of the continuous model equations, i.e.,

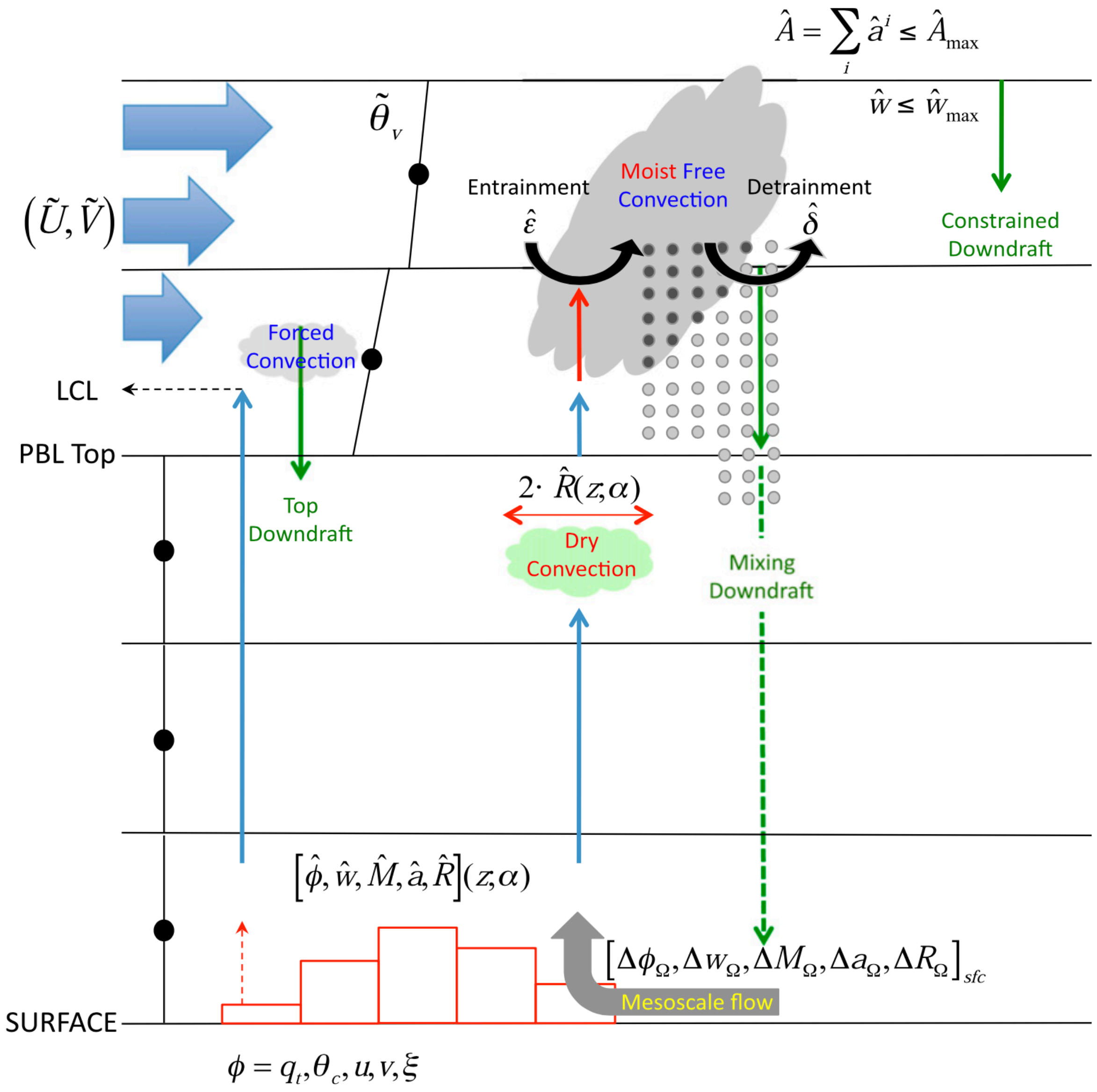
$$\overline{w} = \overline{w}_{model}$$

A Unified Plume Model for Process-Dependent Parameterization



Adapted from Park (2014)

Governing Equations



$$\frac{\partial \bar{\phi}}{\partial t} + \bar{\mathbf{V}} \cdot \nabla \bar{\phi} = -g \frac{\partial}{\partial p} \left[\sum_i \hat{M}^i (\hat{\phi}^i - \tilde{\phi}) + \sum_j \check{M}^j (\check{\phi}^j - \tilde{\phi}) \right] + g \left(\sum_i \hat{M}^i \hat{S}^i_\phi + \sum_j \check{M}^j \check{S}^j_\phi \right) + \tilde{a} \left(\frac{\partial \tilde{\phi}}{\partial t} \right)_s ,$$

$$\frac{1}{\hat{M}} \frac{\partial \hat{M}}{\partial p} = \hat{\epsilon} - \hat{\delta},$$

$$\frac{1}{\check{M}} \frac{\partial \check{M}}{\partial p} = \check{\epsilon} - \check{\delta},$$

$$\frac{\partial \hat{\phi}}{\partial p} = -\hat{\epsilon}(\hat{\phi} - \tilde{\phi}_u) - \hat{\delta}(\hat{\phi}_* - \hat{\phi}) + \hat{S}_\phi + \hat{C}_\phi, \quad \text{and}$$

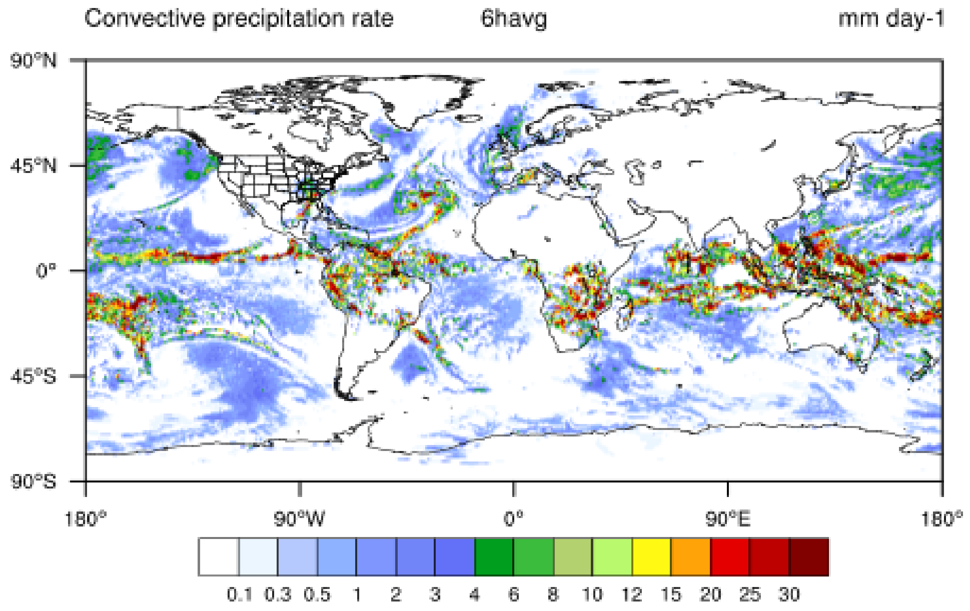
$$\frac{\partial \check{\phi}}{\partial p} = -\check{\epsilon}(\check{\phi} - \tilde{\phi}_d) - \check{\delta}(\check{\phi}_* - \check{\phi}) + \check{S}_\phi + \check{C}_\phi,$$

$$\hat{S}_\phi \equiv \left(\frac{\hat{a}}{g\hat{M}} \right) \left(\frac{\partial \hat{\phi}}{\partial t} \right)_s = \left(\frac{\partial \hat{\phi}}{\partial p} \right)_s, \qquad \hat{C}_\phi \equiv \left(\frac{\hat{a}}{g\hat{M}} \right) \left(\frac{\partial \hat{\phi}}{\partial t} \right)_c = \left(\frac{\partial \hat{\phi}}{\partial p} \right)_c,$$

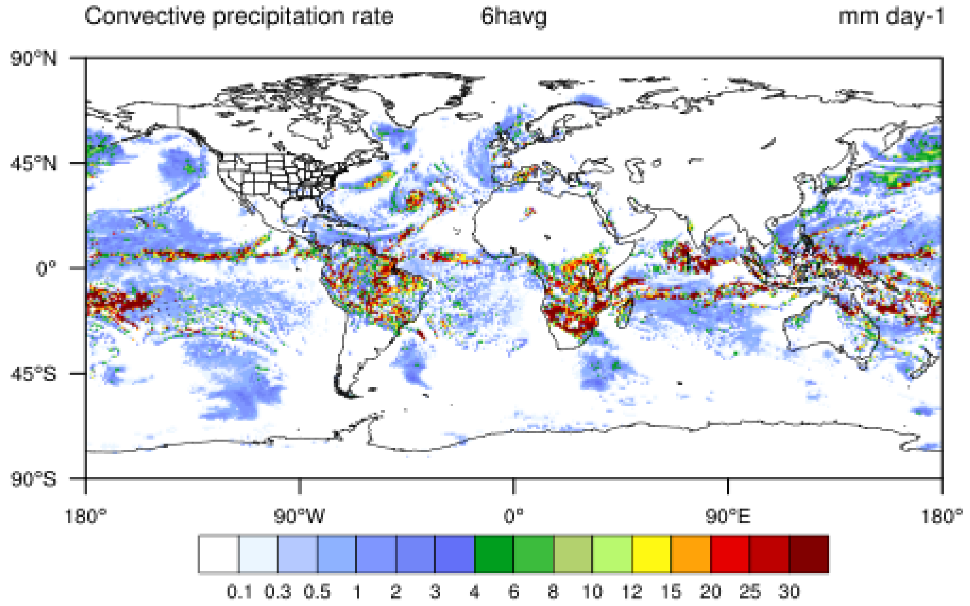
$$\check{S}_\phi \equiv \left(\frac{\check{a}}{g\check{M}} \right) \left(\frac{\partial \check{\phi}}{\partial t} \right)_s = \left(\frac{\partial \check{\phi}}{\partial p} \right)_s \qquad \check{C}_\phi \equiv \left(\frac{\check{a}}{g\check{M}} \right) \left(\frac{\partial \check{\phi}}{\partial t} \right)_c = \left(\frac{\partial \check{\phi}}{\partial p} \right)_c$$

Adapted from Park (2014)

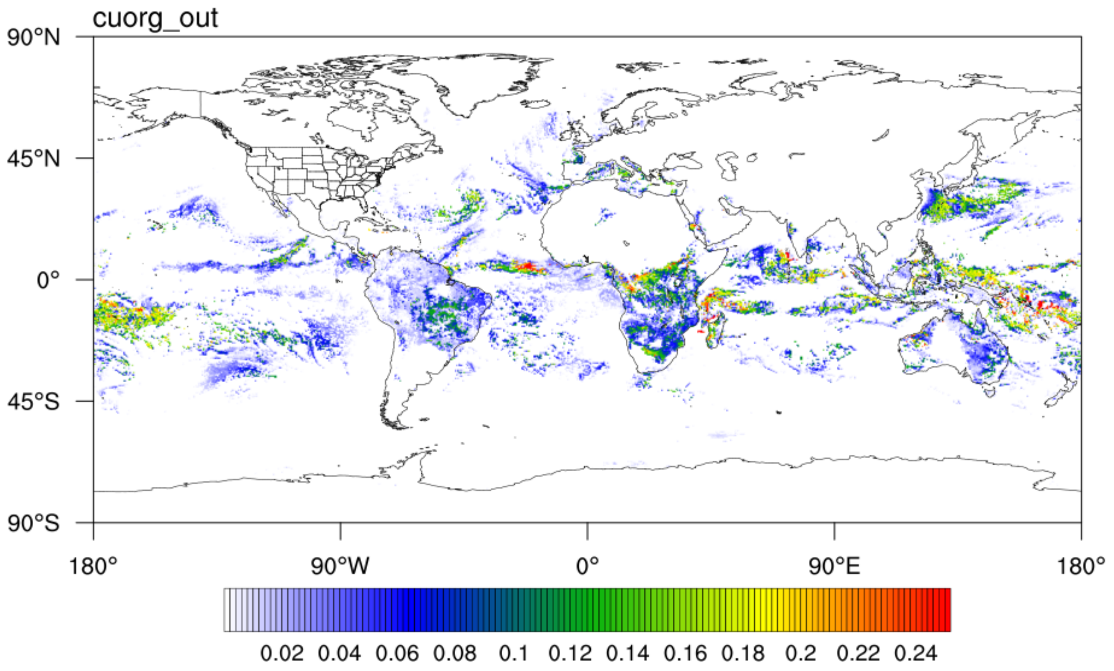
GFSv17P8 Run



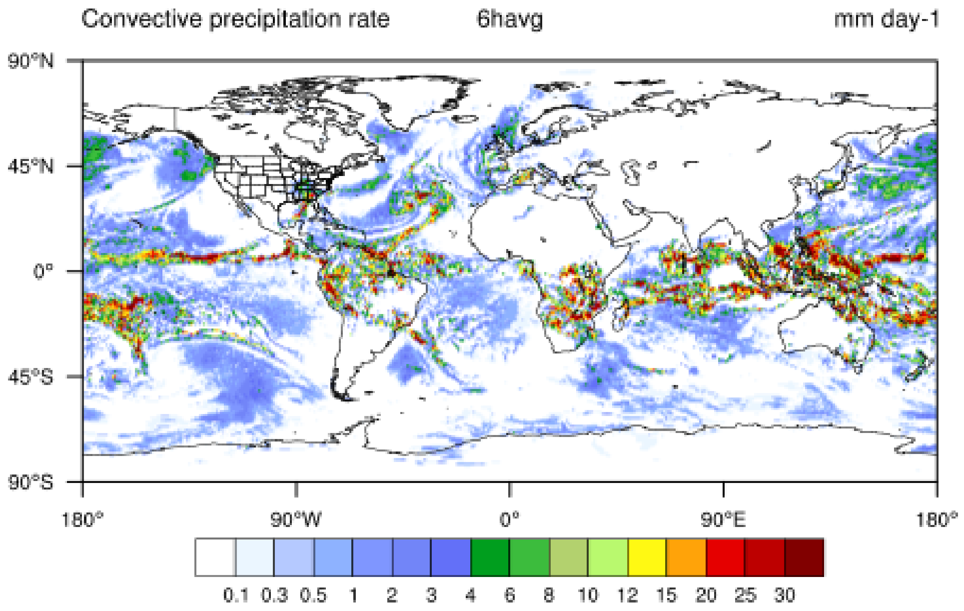
UNICON Run



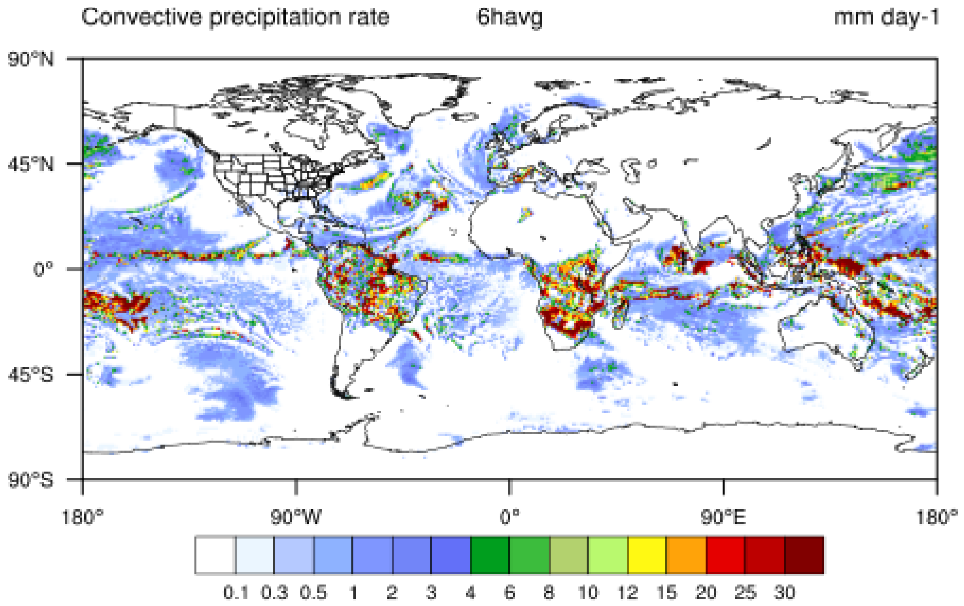
UNICON Run: Cold Pool Fraction



GFSv17P8 Run



UNICON Run



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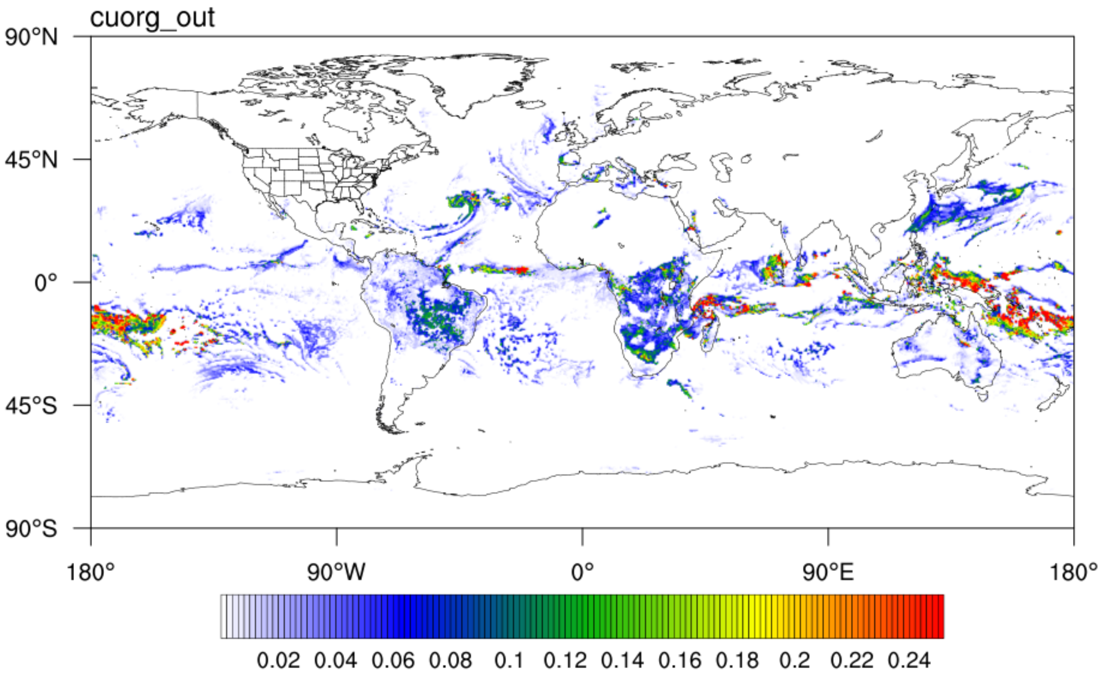
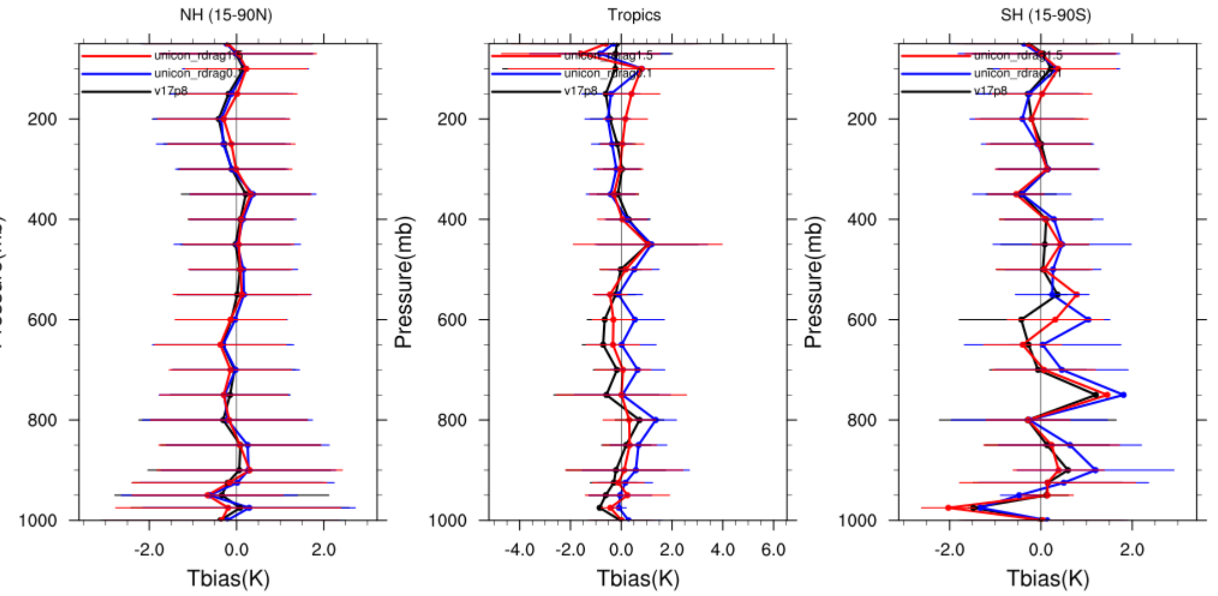
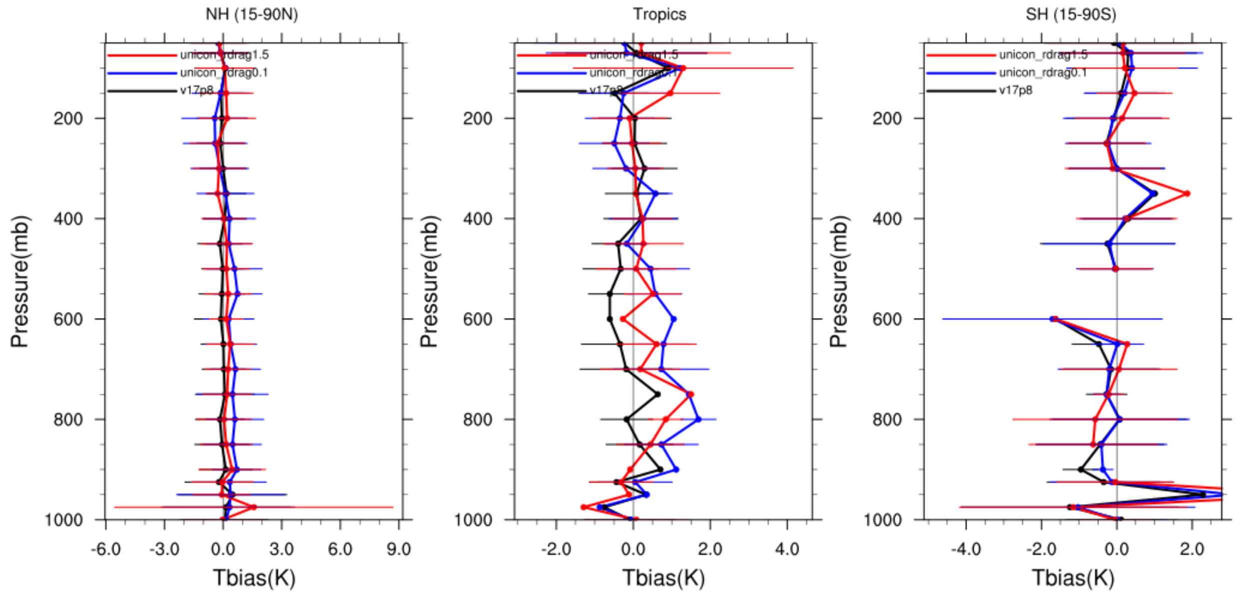


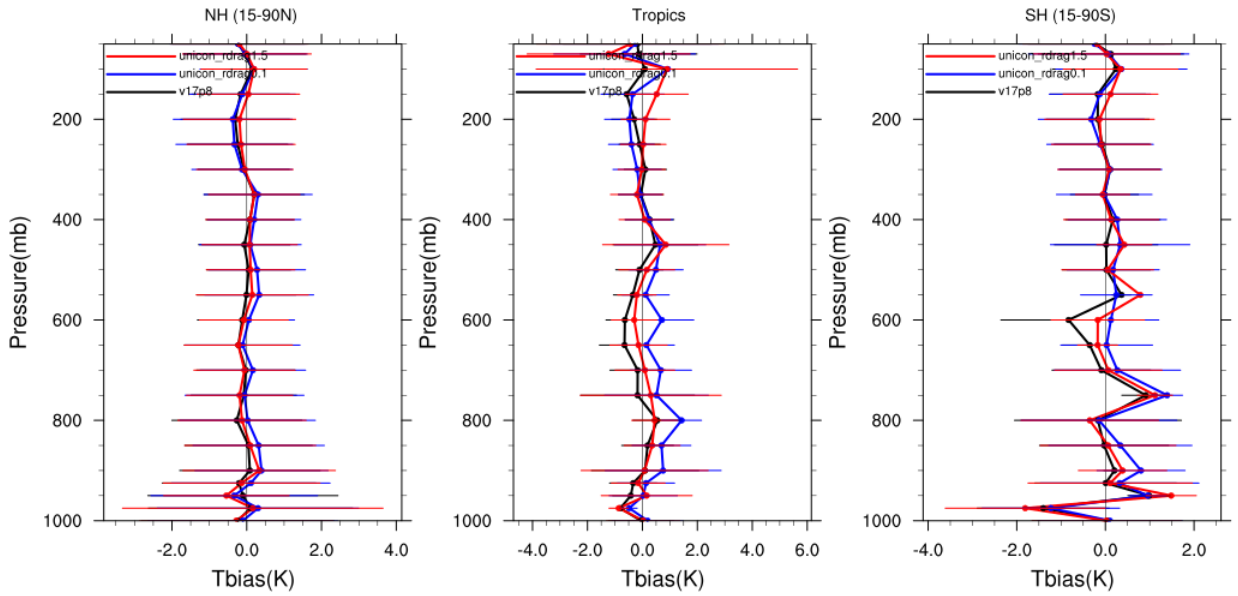
ADUPA Temperature Bias Avg 6 Winter cases for fcst hr 48



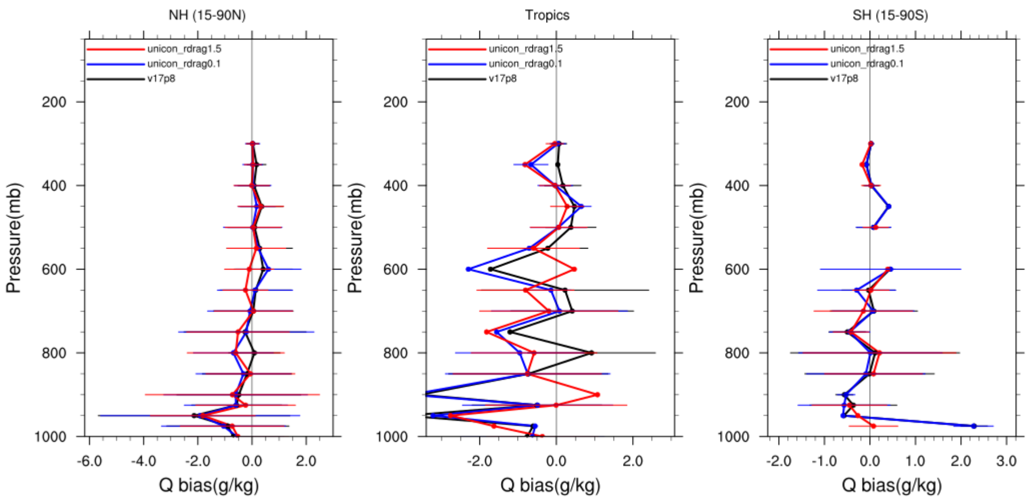
ADUPA Temperature Bias Avg 6 Summer cases for fcst hr 48



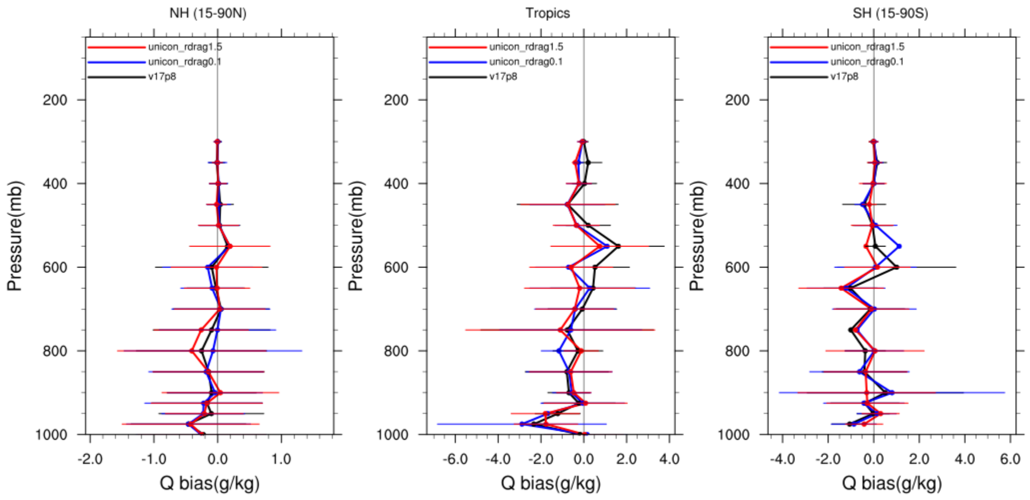
ADUPA Temperature Bias Avg 12 cases for fcst hr 48



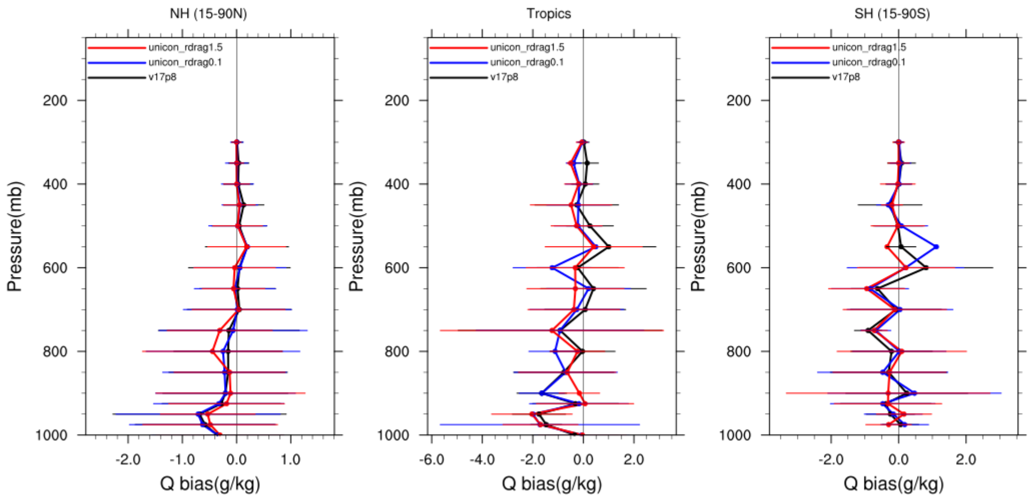
ADUPA Specific Humidity Bias Avg 6 Summer cases for fcst hr 48

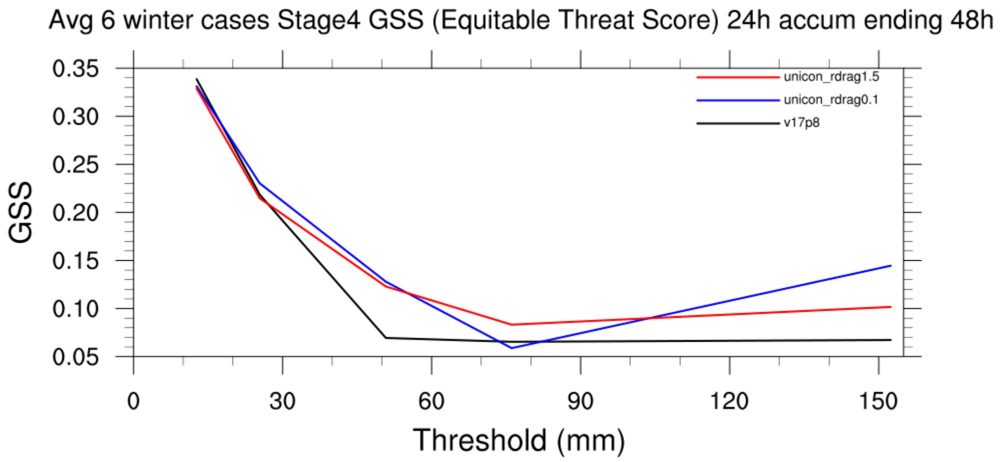


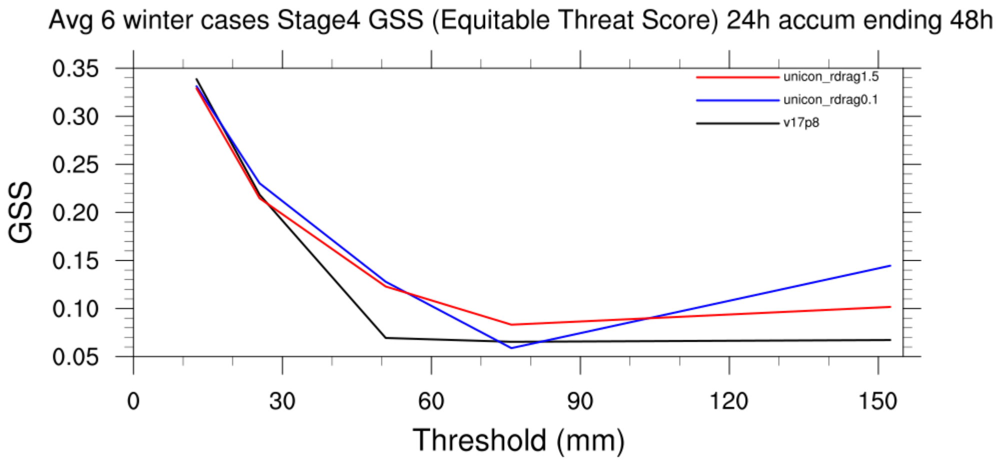
ADUPA Specific Humidity Bias Avg 6 Winter cases for fcst hr 48



ADUPA Specific Humidity Bias Avg 12 cases for fcst hr 48




Avg 6 winter cases Stage4 GSS (Equitable Threat Score) 24h accum ending 48h
unicon_rdrag1.5
unicon_rdrag0.1
v17p8
GSS
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
30
60
90
120
150
Threshold (mm)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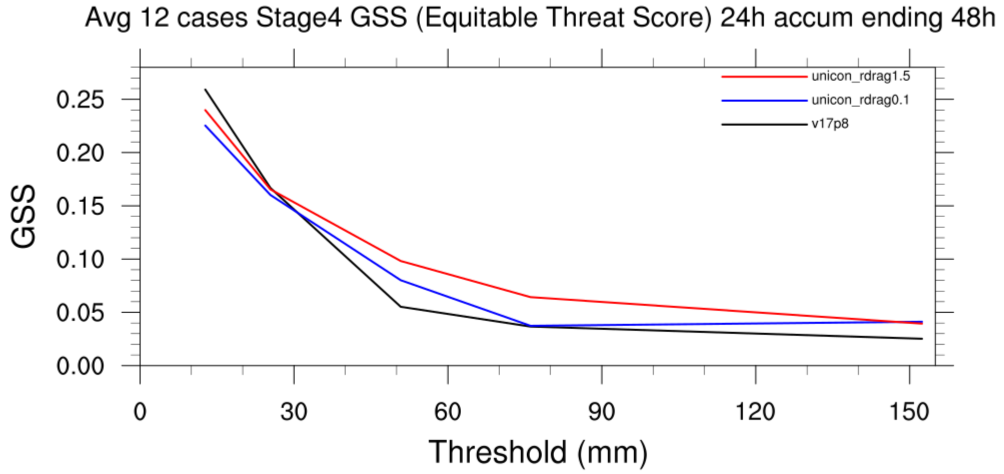
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30
60
90
120
150
Threshold (mm)

Summary

- The separation of the resolved and unresolved scales of motion in atmospheric model development is of utmost conceptual importance. It should reinvigorate our attention for the "gray-zone" physics parameterization development.
- According to the spatial filter framework, unresolved processes relevant to the approximated resolved solution of atmospheric models should be treated as relative to the resolved motion, allowing us to make the parameterization of unresolved processes as simple as possible.
- There is an ambiguity in the approach for implementing scale awareness in the UFS convection scheme (and in any similar scaling convection schemes using the conventional parameterization formulation, as well as in the *ad hoc* separation of deep, shallow and PBL convection in the UFS.
- A pathway forward to circumvent the ambiguity and unify the representation of subgrid convection in the UFS is to use the unified scale-adaptive approach developed by Park (2014) based on a unified plume model.
- The experiment using the unified approach in the UFS shows its promising potential as a research and development direction.

Thank You!