IS WEATHER CHAOTIC? COEXISTING ATTRACTORS AND MULTISTABILITY

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摘 要

自勞倫茲 1963 年開創性有限預報度的研究和 1972 年巧妙蝴蝶效應的比喻以來，“天氣是混沌〞的說法已被廣泛接受。這種觀點將我們的注意力，從拉普拉斯的決定論觀點，強調的是規律性，轉向於混沌的不規律性。自 1963 年以來，這樣的觀念已主宰超過半個世紀。然而，勞倫茲模式中的混沌解具有排它性，即混沌解的出現，意謂著規律解不可能存在。相較下，在完成延伸勞氏模式成為廣義勞倫茲模式的研究中，作者和共同作者 (Shen 2019a, b; Shen et al. 2019) 強調混沌和規律解的共存性，稱之為共存吸引子。亦即在使用相同的模式和參數下，混沌解和非混沌解都有可能出現。而他們的出現決定於初始條件。這樣的結果顯示：複雜的天氣具有混沌和秩序的雙重性質，而這兩者具有明顯不同的可預報度。近期，我們更進一步指出以下兩種可以產生或調變共存吸引子的機制：(1) 小尺度對流匯總的負反饋。它使穩定解得以出現，而且該解可以與混沌或非線性振盪解共存；(2) 大尺度隨時間變化的作用力 (如加熱) 的調變 (Shen et al. 2021a, b)。

最近，為了重申勞倫茲模式中的結果，可以有效應用或說明現實世界中的問題。作者提供了勞倫茲模式和 Pedlosky 模式中數學的普遍性，亦即兩個模式的常微分方程組可以完全相同。同時，作者也指出存在非耗散勞倫茲模式、Duffing、非線性薛丁格和 Korteweg-de Vries 方程式之間的普遍性 (Shen 2020, 2021)。此外，我們也比較了勞倫茲 1963 和 1969 的模式。前者是具有單穩定態的有限尺度、非線性、混沌模式，而後者是基於閉合假設的(closure based)，在物理上具多重空間尺度的，數學上是線性的，而數值上具有病態的模式。為了支持和說明我們對天氣本質修正後的觀點，即混沌和秩序的雙重性質，我們最近的一份簡短報告 (Shen et al. 2021c) 使用滑雪和泛舟作為類比，而加以闡述了單穩定態和多重穩定態的差異。而本報告進一步延伸相關的研究，以期了解修訂後的觀點對真實世界數值預測和天氣分析的影響。我們將使用颶風軌跡預測來進行說明，並且總結近來發展的分析方法，包括多重尺度交互作用的分析方法，迴歸分析方法，混沌和非混沌解的分類方法。

關鍵字：共存吸引子，混沌，廣義勞倫茲模式，預報度, 單穩定態, 多重穩定態

ABSTRACT

Since Lorenz's 1963 study and 1972 presentation, the statement "weather is chaotic'' has been well accepted. Such a view turns our attention from regularity associated with Laplace's view of determinism to irregularity associated with chaos. In contrast to single type chaotic solutions, recent studies using a generalized Lorenz model (Shen 2019a, b; Shen et al. 2019) have focused on the coexistence of chaotic and regular solutions that appear within the same model, using the same modeling configurations but different initial conditions. The results suggest that the entirety of weather possesses a dual nature of chaos and order with distinct predictability. Furthermore, Shen et al. (2021a, b) illustrated the following two mechanisms that may enable or modulate attractor coexistence: (1) the aggregated negative feedback of small-scale convective processes that enable the appearance of stable, steady-state solutions and their coexistence with chaotic or nonlinear limit cycle solutions; and (2) the modulation of large-scale time varying forcing (heating).

Recently, the physical relevance of findings within Lorenz models for real world problems has been reiterated by providing mathematical universality between the Lorenz simple weather and Pedlosky simple ocean models, as well as amongst the non-dissipative Lorenz model, and the Duffing, the Nonlinear Schrodinger, and the Korteweg–de Vries equations (Shen 2020, 2021). We additionally compared the Lorenz 1963 and 1969 models. The former is a limited-scale, nonlinear, chaotic model; while the latter is a closure-based, physically multiscale, mathematically linear model with ill-conditioning. To support and illustrate the revised view, a recent short report (Shen et al. 2021c) elaborated on additional details of monostability and multistability by applying skiing and kayaking as an analogy. To address the influence of the revised view on real-world model predictions and analysis, as illustrated using hurricane track predictions, and to provide a summary on the recent deployment of methods for multiscale analyses and classifications of chaotic and non-chaotic solutions, this report further extends recent studies.

Keywords: attractor coexistence, chaos, generalized Lorenz model, predictability, monostability, multistability

INTRODUCTION

Two studies of Prof. Lorenz (Lorenz 1963, 1972) laid the foundation of chaos theory that emphasize a Sensitive Dependence of Solutions on Initial Conditions (SDIC). While the concept of SDIC can be found in earlier studies (e.g., Poincare 1890), the rediscovery of SDIC in Lorenz (1963) changed our view on the predictability of weather and climate, yielding a paradigm shift from Laplace's view of determinism with unlimited predictability to Lorenz's view of deterministic chaos with finite predictability. Based on an insightful analysis of the Lorenz 1963 and 1969 (L63 and L69) models, as well as the recent development of generalized Lorenz models (GLM, Shen 2014, 2019a,b; Shen et al. 2019), such a conventional view is being revised to emphasize the dual nature of chaos and order given recent studies (Shen et al. 2021a,b). To support and illustrate the revised view, a recent short report (Shen et al. 2021c) describes additional details for the following features: (1) Continuous vs. Sensitive Dependence on Initial Conditions (CDIC vs. SDIC); (2) single-types of attractors and monostability within the L63 model; (3) coexisting attractors and multistability within the GLM; (4) skiing vs. kayaking: an analogy for monostability and multistability; and (5) a list of non-chaotic weather systems.

By first reviewing the above features of (4) and (5), this report further extends recent studies to address the influence of the revised view on real-world model predictions and analyses by (a) viewing chaotic and non-chaotic solutions as steering flows in order to illustrate their impact on track predictions; (b) distinguishing instability, chaos, and computational chaos; (c) revealing saturation dependence on various types of solutions; and (d) providing a summary on the recent deployment of methods for analyzing scale interaction and detecting multistability.

ANALYSIS AND DISCUSSION

Monostability and multistability illustrated using skiing and kayaking

To illustrate SDIC, Lorenz (1993) applied the activity of skiing (left in Fig 1) and developed an idealized skiing model for revealing the sensitivity of time-varying paths to initial positions (middle in Fig 1). Based on the left panel, monostability appears when slopes are steep everywhere. Namely, SDIC always appear. In comparison, the right panel for kayaking can be used to illustrate multistability. In the photo, the appearance of strong currents and a stagnant area (outlined with a white box) suggest instability and local stability, respectively. As a result, when two kayaks move along strong currents, their paths display SDIC. On the other hand, when two kayaks move into the stagnant area, they become trapped, showing no SDIC. Such features of SDIC or no SDIC illustrate the nature of multistability.

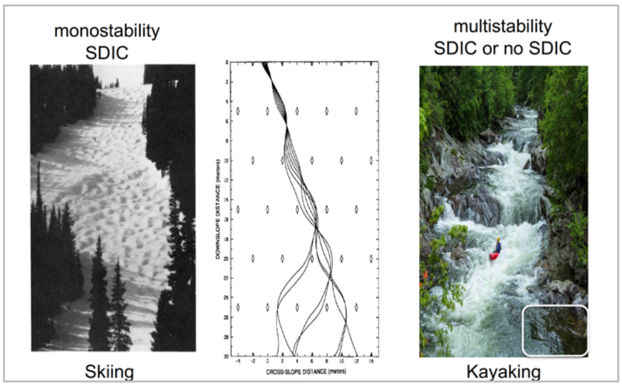

Fig 1. Skiing as used to reveal monostability (left and middle, Lorenz 1993), and kayaking as used to indicate multistability (right, Copyright: ©Carol-stock.adobe.com)

Non-chaotic weather systems

The concept of multistability suggests the possibility for coexisting chaotic and non-chaotic weather systems. Non-chaotic solutions have been previously applied for understanding the dynamics of different weather systems, including steady-state solutions for investigating atmospheric blocking (e.g., Charney and DeVore 1979; Crommelin et al. 2004), limit cycles for studying 40-day intra-seasonal oscillations (Ghil and Robertson 2002), Quasi-Biennial Oscillations (e.g., Renaud et al. 2019) and vortex shedding (Ramesh et. al. 2015), and nonlinear solitary-pattern solutions for understanding morning glory (i.e., a low-level roll cloud, Goler and Reeder 2004). Below, we present how a chaotic or non-chaotic, steady-state solution may be viewed as a "steering" flow to illustrate its impacts on the movement of a tropical cyclone (TC).

Chaotic and non-chaotic solutions as steering flows

Three types of steering flows (associated with a saddle, a spiral source, or a spiral sink) are presented below and two kinds of track errors (Ivan-type vs. Sandy-type) are classified.

As discussed in Shen et al. (2021c), a chaotic solution displays both CDIC and SDIC (Fig. 2a), corresponding to the "regular" oscillation associated with a spiral source and the "irregular" oscillation associated with a saddle point (Fig. 2b), respectively. A zoomed-in view for the spiral source and saddle point is provided in Figs. 3a and 3b, respectively. Although both points are unstable within the Lorenz 1963 model, the saddle point provides an essential ingredient for chaos. A trajectory near the spiral source may "regularly" move until it shifts away from the spiral source and towards the saddle point. Therefore, within the Lorenz 1963 model, a chaotic solution may display regular or irregular oscillations within short time intervals, depending on its location (i.e., near the spiral source or the saddle point). By comparison, the GLM allows for the coexistence of a stable spiral sink (Fig. 3c) and the saddle point. Hence, when a trajectory initially begins near the spiral sink, it may behave regularly during its entire lifetime. Such a scenario may occur when a kayak begins in a stagnant region (e.g., Fig 1).

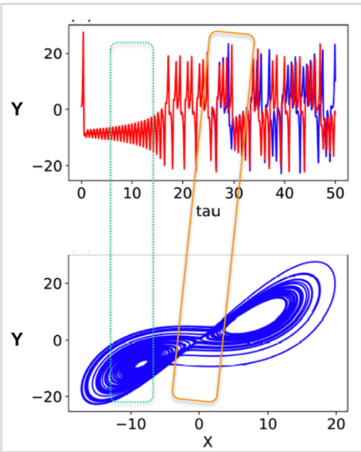

Fig. 2: A time-varying chaotic solution within the Lorenz 1963 model (top) and its two-dimensional phase portrait (bottom). A green (orange) box indicates the association of regular (irregular) oscillation with a spiral source (a saddle).

Figure 3 displays three types of basic flows, including a spiral source, a saddle, and a spiral sink (from left to right). By viewing the "solutions" in Fig. 3 as steering flows, the above discussions suggest that flows associated with a spiral source or a spiral sink may lead to incremental changes of TC movement (or to incremental bias for TC track prediction), while a saddle point may lead to rapid changes of TC movement. From a perspective of steering flows, two types of TC track errors, as shown in Fig. 4, include: (i) an Ivan (2004)-type with a persistent track bias associated with an underestimated sub-tropical ridge (Stewart 2004; Shen et al. 2006) and (ii) a Sandy (2012) type with rapid diverged tracks associated with a steering flow that contains a saddle point[1], respectively (Blake et al. 2013; Shen et al. 2013). Very slight differences determine whether a TC (e.g., Sandy) recurves to the northeast, or wraps back west.

Therefore, improving these track predictions requires an analysis of the location and intensity (i.e., intensification or weakening) of a subtropical ridge and/or "predicting" the potential for the appearance of a saddle point, (e.g., whether two large-scale systems that move in an opposite direction may approach one other).

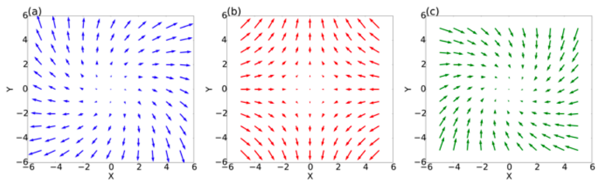

Fig. 3: Three basic types of solutions, including a spiral source (a), a saddle (b), and a spiral sink (c).

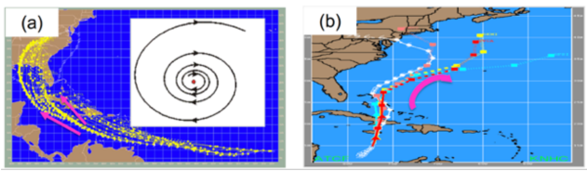

Fig. 4: Two types of track errors associated with different steering flows. (a) Hurricane Ivan (2004) (e.g., Stewart 2004) and (b) Hurricane Sandy (2012) (e.g., Blake et al. 2013).

Instability, Chaos, and Computational Chaos

Simple definitions of instability and chaos are defined as follows: (1) instability is defined as an unbounded amplification, and (2) chaos is defined as a bounded time-varying growing solution that requires solution boundedness and, at least, one positive Lyapunov exponent (LE, Wolf et al. 1985; Shen 2014, 2019a). Such a definition with a positive LE and boundedness is consistent with the definition of chaos based on SDIC.

Strictly speaking, instability (or stability) dominates when an orbit is near the spiral source (or the spiral sink).

[1] Sandy's sinuous track with a northwestward turn prior to its landfall are very likely due to the complicated multiscale interactions of Sandy with its environmental flows, such as upper-level troughs in the westerly jet stream and a blocking pattern to the west and east of Sandy (Blake et al. 2013).

Such dynamics are less complicated as compared to chaotic dynamics. Namely, better predictability is expected. Chaotic dynamics become important or dominate when an orbit moves closer to a saddle point. (Note that the above discussions are based on two-dimensional, unstable, spiral critical points. Within the three-dimensional phase space, the unstable non-trivial critical point may contain a 2D spiral source and a stable manifold in the 3rd dimension, appearing as a special kind of saddle that complicates the dynamics and, thus, discussions).

In Lorenz (1989, 2006), the term computational chaos was introduced for indicating the appearance of chaotic responses associated with large time steps. Such a feature can be illustrated using the logistic equation (Eq. 1) and the logistic map (Eqs. 2a-2c), as shown below:

$$dX/d\tau = rX\,(1 - X), \qquad (1)$$

$$Y_{n+1} = \rho Y_n\,(1 - Y_n), \qquad (2a)$$

$$Y_n = r\Delta\tau X_n/(1 + r\Delta\tau), \qquad (2b)$$

$$\rho = 1 + r\Delta\,\tau. \qquad (2c)$$

Here, τ represents the time variable and $\Delta\tau$ the time step. The two time-dependent variables are X and Y, and the two time-independent parameters are r and ρ. Eq. (2) is obtained from Eq. (1) using a forward finite difference scheme. Therefore, while Eq. (1) is continuous in time, Eq. (2) is discrete in time. As summarized in Table 1, Eq. (1) contains an analytical, non-chaotic solution and Eq. (2) produces bifurcation and chaos at a large parameter, ρ, requiring a large $\Delta\tau$ as a result of Eq. (2c). Therefore, "irregular responses" in Eq. (2) may be viewed as computational chaos. Similarly, such a feature of bifurcation was previously documented using a discrete version of the equation for terminal velocities (e.g., Shen and Lin 1995).

Table 1: Computational chaos illustrated using the Logistic eq. and Logistic map in Eqs. (1) and (2).

Name	Type	Solution	Eq.
Logistic Eq.	differential	analytical, non-chaotic	(1)
Logistic map	difference	chaotic at large time steps	(2)

Saturation dependence on various types of solutions

Within nonlinear chaotic solutions, root-mean-square (RMS) forecast errors may approach constants as time proceeds, being saturated when sufficiently large ensemble runs are applied. Since all of the steady-state solutions eventually become constant, their RMS errors may appear saturated. In contrast, nonlinear oscillatory solutions may produce the oscillatory RMS errors (e.g., Liu et al. 2009). On the other hand, nonlinear oscillatory solutions may appear as computational chaos, displaying saturation, when insufficient temporal solutions are used. Therefore, saturated RMS errors should not be used as a sole indicator for revealing the chaotic nature of weather.

Methods for analyzing scale interaction and detecting multistability

The above discussions suggest that an effective detection of scale modulation and/or non-chaotic solutions may lead to better predictability, improving our confidence in numerical weather and climate predictions. In our recent studies, in addition to the "standard" method for computing Lyapunov Exponents (LEs) within the 5D-9D Lorenz models (Shen 2014, 2019a), the following methods have been deployed: (1) the Parallel Ensemble Empirical Mode Decomposition (PEEMD) for revealing scale interactions (Wu and Shen 2016; Shen et al. 2017); (2) Recurrence Plots (RPs) for the analysis of multiple African easterly waves that display differences in phases and amplitudes (Reyes and Shen 2019; 2020); and (3) a Kernel Principal Component Analysis (K-PCA) for separating chaotic and non-chaotic attractors (Cui and Shen 2021).

CONCLUDING REMARKS

An insightful analysis of the classical Lorenz 1963 and 1969 models and the development of the generalized Lorenz model (GLM) recently suggested the following:

- The Lorenz 1963 nonlinear model with monostability is effective for revealing the chaotic nature of weather, suggesting finite intrinsic predictability.
- The Lorenz 1969 model is a closure-based, physically multiscale, mathematically linear model with ill-conditioning; it easily captures numerical instability and, thus, is effective for revealing finite predictability.
- The GLM with coexisting attractors and multistability suggests both limited and unlimited intrinsic predictability.
- Using selected cases within a global model (e.g., Shen 2019b), a practical predictability of 30 days was previously documented.

Based on the above results, we previously proposed a revised view on the dual nature of chaos and order with distinct predictability in weather and climate.

To support and illustrate the revised view, a recent report (Shen et al. 2021c) elaborated on additional details of the monostability and multistability by applying skiing and kayaking as an analogy. This report further extends related studies in order to address the influence of the revised view on real-world model predictions and analyses. By viewing chaotic and non-chaotic solutions as steering flows, we identified two-types of track errors, including Ivan (2004)-type and Sandy (2012)-type, whose movements were impacted by a spiral source (as a subtropical ridge) and a saddle point, respectively. The former should be more predictable.

We additionally discussed differences surrounding instability, chaos, and computational chaos, and illustrated the saturation dependence on various types of solutions. While saturated RMS errors may indicate either chaos or computational chaos, oscillatory RMS errors are likely associated with nonlinear oscillatory solutions. We finally provided a summary on the recent deployment of methods (e.g., PEEMD, RP, and K-PCA) for multiscale analyses and classifications of chaotic and non-chaotic solutions with the aim of identifying systems with better predictability.

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