Hybrid Mass Flux Cumulus Schemes (HYMACSs) as a Step to Unified Cumulus Parameterization and Its Application to Tropical Cyclone Intensity Prediction

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Abstract

Cumulus parameterization (CP) embodies scientists' understanding in the interaction between cumulus eddies and environmental mean fields, but there is a gray zone (3 km – 20 km spatial scale) between the scale of eddies and mean fields. The unified CP aims to cover the gray zone. Previous studies showed that Kain-Fritsch schemes (KFSs) are the best CP schemes in terms of tropical cyclone prediction, but KFSs overestimate the intensity of tropical cyclones in the gray-zone scale.

This study introduces hybrid mass flux cumulus schemes (HYMACSs), which solve convective drafts implicitly but compensating motions explicitly by a mass source/sink term in the continuity equation. It is hypothesized that the horizontally limited compensating subsidence in the KFS overheats a grid point, which further overestimates the intensity of a tropical cyclone. To verify the hypothesis, experiment is ongoing.

Key word: hybrid mass flux cumulus scheme, tropical cyclone intensity prediction

1. Introduction

Scientifically, cumulus parameterization (CP) is an important multi-scale issue because it embodies scientists' understanding in the interaction between cumulus eddies and environmental mean. However, there is a gray zone between the scale of eddies and mean. Molinari and Dudek (1992) summarized that for horizontal grid spacing smaller than 3 km, models solve cumulus explicitly, and for that larger than 20 km, models solve cumulus effects implicitly by CP. To close the gap between these, Arakawa and Wu (2013) launched the unified CP problem, and pointed out that a key to the problem is σ , the fractional area covered by convective drafts in a grid cell. σ is assumed to be negligible conventionally (hereafter, the term "conventional" refers to this assumption), but becomes important while the horizontal grid spacing shrinks toward 3 km. Moreover,

our study is going to propose another possible key to the unified CP problem.

Technologically, classical mass flux type CP schemes, especially Kain-Fritsch schemes (Kain and Fritsch, 1990, 1993; Kain, 2004. KFSs), are widely used in numerical weather prediction models. Sensitivity experiments showed that KFSs are the best CP schemes in terms of tropical cyclone intensity or track prediction (e.g., Prater and Evans, 2002; Rao and Prasad, 2007; Srinivas et al., 2007; Chandrasekar and Balaji, 2012; others suggested using Grell scheme, e.g., Mandal et al., 2004; Yang and Ching, 2005), but models applying KFSs with a horizontal grid spacing about 10 km tend to overestimate the intensity of tropical cyclones (e.g., Srinivas et al., 2007; Chandrasekar and Balaji, 2012; Sun et al., 2013; Singh and Mandal, 2014; Haghroosta et al., 2014). This implies a systematic bias of KFSs in the

gray-zone scale.

Touching the reason for the systematic bias, classical mass flux type CP schemes assume that convective drafts are always accompanied by compensating motions which exactly cancel out the convective mass fluxes in the grid point (hereafter, the term "classical" refers to this assumption). Conceptually, this assumption is valid only for models with a horizontal grid spacing larger than the Rossby radius of deformation. Otherwise, the validity of it becomes questionable because the compensating motions turn resolvable and should not be parameterized.

Consequently, the horizontally limited compensating subsidence may over-heat a grid point with cumulus activities, which may further overestimate the intensity of tropical cyclones.

As for the solution, Kain and Fritsch (1993) suggested as a future objective for KFSs that compensating motions may be solved explicitly by introducing a convective mass source/sink term to the resolvable-scale continuity equation in a fully compressible model. Then, Kuell et al. (2007) realized this approach and proposed a new type of CP schemes called hybrid mass flux cumulus schemes (HYMACSs), in which convective mass fluxes are solved not only implicitly for convective drafts but also explicitly for compensating motions. As an advantage of HYMACSs, they showed that mass exchange between the model grid and the parameterization scheme is independent of the horizontal grid spacing from 3.5 km to 28 km. This implies that HYMACSs are a solution to the gray-zone scale problem. Moreover, Kuell and Bott (2008) found their HYMACS to be more realistic than the KFS and the Tiedtke scheme in an idealized simulation of a sea breeze circulation initiating convection.

It is hypothesized that implementing HYMACSs could be as pivotal to the unified CP problem as parameterizing σ , because as the horizontal grid spacing shrinks from 102 km to 100 km, it becomes too much smaller than the Rossby radius of deformation, and then

becomes too small to neglect σ . On the other hand, applying HYMACSs might be a better way to simulate tropical cyclone intensification than using KFSs while the horizontal grid spacing is about 10 km. Section 2 introduces the methodology of HYMACSs, and proposes a new HYMACS, hereafter the HYMAKFS, adapted from the modified KFS in the weather research and forecasting model (WRF), version 3.6.1 (Skamarock et al., 2008). Section 3 describes the ongoing projects and gives a summary.

2. Methodology

A. Ensemble mean continuity equations

Cumulus convection is a turbulent flow. To represent the uncertainty of a turbulent flow, a prognostic variable is decomposed into an ensemble mean component resolvable, and a turbulent eddy component unresolvable; the former is the matter of concern, and the latter is important due to its nonlinear interaction with the former. After the decomposition, the continuity equation becomes:

Since the mean component is what concerns us, we take the ensemble mean of (1) and get the ensemble mean continuity equation:

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{z} + \nabla_{\mathbf{z}} \cdot (\overline{\rho \psi} \overline{\mathbf{V}}) + \frac{\partial}{\partial z} (\overline{\rho \psi} \overline{w}) + \nabla_{\mathbf{z}} \cdot [\overline{(\rho \psi)' \mathbf{V}'}] + \frac{\partial}{\partial z} [\overline{(\rho \psi)' w'}] = \overline{\rho} \overline{\psi} + \overline{\rho' \psi'}.$$
(2)
If we take the ψ , in (2) as unity, we get the ensemble

If we take the ψ in (2) as unity, we get the ensemble mean mass continuity equation:

$$\left(\frac{\partial \bar{\rho}}{\partial t}\right)_{z} + \nabla_{\mathbf{z}} \cdot (\bar{\rho} \overline{\mathbf{V}}) + \frac{\partial}{\partial z} (\bar{\rho} \overline{w}) + \nabla_{\mathbf{z}} \cdot (\bar{\rho}' \overline{\mathbf{V}'}) + \frac{\partial}{\partial z} (\bar{\rho}' w') = 0.$$
(3)

For the coupling to the WRF, (3) is transformed from height coordinate to the terrain-following hydrostatic-pressure coordinate so called η coordinate.

B. General closure of hybrid mass flux cumulus schemes (HYMACSs)

Closure assumptions are required to diagnose the horizontal eddy flux per area $\overline{(\rho\psi)'\mathbf{V}'}$, the vertical eddy flux per area $\overline{(\rho\psi)'w'}$ and the eddy source/sink per volume $\overline{\rho'\dot{\psi}'}$ from resolvable prognostic variables in order to close the equation set of fluid dynamics. To focus on the to-be-diagnosed terms in (2), define:

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_z + \nabla_{\!\mathbf{z}} \cdot (\overline{\rho \psi} \overline{\mathbf{V}}) + \frac{\partial}{\partial z} (\overline{\rho \psi} \overline{w}) - \bar{\rho} \bar{\psi} = \overline{\rho' \dot{\psi}'} - \nabla_{\!\mathbf{z}} \cdot$$

$$\left[(\rho \psi)' \mathbf{V}' \right] - \frac{\partial}{\partial z} \left[(\rho \psi)' w' \right] \equiv \left(\frac{\partial \overline{\rho \psi}}{\partial t} \right)_{cumulus}. \tag{4}$$

Diagnosing $\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{cumulus}$ is the goal of the general closure of HYMACSs. Accordingly, rewrite (4) by the

definition of ensemble mean as:

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{cumulus} = \left[\overline{\rho \psi} - \nabla_{\mathbf{z}} \cdot (\overline{\rho \psi \mathbf{V}}) - \frac{\partial}{\partial z} (\overline{\rho \psi w})\right] -$$

$$\left[\overline{\rho}\overline{\dot{\psi}} - \nabla_{\mathbf{z}} \cdot \left(\overline{\rho\psi}\overline{\mathbf{V}}\right) - \frac{\partial}{\partial z} \left(\overline{\rho\psi}\overline{w}\right)\right]. \tag{5}$$

For generality, following Anthes (1977), ensemble mean is assumed to be spatially homogeneous in a grid box so that ensemble mean is equal to grid-box mean, thus ensemble mean terms in (5) become grid-box mean:

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{cumulus} \cong \left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{cumulus'} = \frac{1}{v} \left\{ \iiint_{V} \left[\rho \dot{\psi} - \nabla_{\!\mathbf{z}} \cdot \right] \right\}$$

$$(\rho\psi\mathbf{V}) - \frac{\partial}{\partial z}(\rho\psi w) \Big] dV' - \iiint_{V} \Big[\bar{\rho}\bar{\psi} - \nabla_{\mathbf{z}} \cdot (\overline{\rho\psi}\overline{\mathbf{V}}) - \frac{\partial}{\partial z}(\overline{\rho\psi}\overline{w}) \Big] dV' \Big\}, \tag{6}$$

where V is the volume of the grid box. Then, apply the top-hat method; due to the vertical directivity of convective drafts, the first integral in (6) is separated into a few sub-grid-column integrals, a few columns of convective drafts (subscript i) embedded in a column of

environment (tilde):

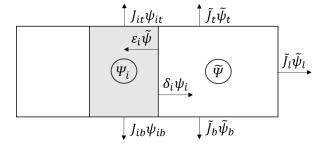
$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{cumulus'} = \frac{1}{v} \left\{ \sum_{i} \iiint_{V_{i}} \left[\rho_{i} \dot{\psi}_{i} - \nabla_{\mathbf{z}} \cdot (\rho_{i} \psi_{i} \mathbf{V}_{i}) - \right] \right\}$$

$$\frac{\partial}{\partial z}(\rho_i\psi_iw_i)\Big]\,dV'+\iiint_{\widetilde{V}}\,\Big[\widetilde{\rho}\widetilde{\psi}-\nabla_{\!\mathbf{z}}\cdot\big(\widetilde{\rho}\widetilde{\psi}\widetilde{\mathbf{V}}\big)\,-$$

$$\frac{\partial}{\partial z} \left(\tilde{\rho} \tilde{\psi} \tilde{w} \right) \right] dV' - \iiint_{V} \left[\bar{\rho} \bar{\psi} - \nabla_{\mathbf{z}} \cdot (\overline{\rho \psi} \overline{\mathbf{V}}) - \frac{\partial}{\partial z} (\overline{\rho \psi} \overline{w}) \right] dV' \right\}, \tag{7}$$

where $V = \tilde{V} + \sum_i V_i$. Next, apply divergence theorem to (7); after Bjerknes (1938), sub-cloud structure and sub-environment structure are assumed to be negligible so that the mass coupled ψ flux is the mass flux multiplied by the ψ . This yields the general closure of HYMACSs:

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{cumulus'} \cong \left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{HYMACS} = \frac{1}{v} \left\{ \sum_{i} \left[\Psi_{i} + \varepsilon_{i} \tilde{\psi} - \delta_{i} \psi_{i} - J_{it} \psi_{it} - J_{ib} \psi_{ib} \right] + \left[\widetilde{\Psi} + \sum_{i} \left(-\varepsilon_{i} \widetilde{\psi} + \delta_{i} \psi_{i} \right) - \widetilde{J}_{l} \widetilde{\psi}_{l} - \widetilde{J}_{t} \widetilde{\psi}_{t} - \widetilde{J}_{b} \widetilde{\psi}_{b} \right] - \left[\overline{\Psi} - \overline{J}_{l} \overline{\psi}_{l} - \overline{J}_{t} \overline{\psi}_{t} - \overline{J}_{b} \overline{\psi}_{b} \right] \right\},$$
 (8) where Ψ , ε , δ and J are respectively source/sink, entrainment rate, detrainment rate and outward mass flux on the boundary of the grid box. The subscript l , t and b denote respectively the lateral, top and bottom boundary of the grid box. Both entrainment rate and detrainment rate are positive definite; the former is the mass flux from the environment to the convective drafts, and the latter is opposite in definition to the former. The three pairs of square brackets represent the mass coupled ψ tendency in respectively the convective drafts, the environment and the mean (Fig. 1). If ψ is taken as



unity, (8) becomes:

Fig. 1. A schematic diagram for (8), a vertical cross-section in a grid box. The gray part is the ith convective draft, and the white part is the environment. The circles are sources/sinks, and the arrows are fluxes.

$$\left(\frac{\partial \overline{\rho}}{\partial t}\right)_{HYMACS} = \frac{1}{V} \left\{ \sum_{i} [\varepsilon_{i} - \delta_{i} - J_{it} - J_{ib}] + \left[\sum_{i} (-\varepsilon_{i} + \varepsilon_{i}) \right] \right\}$$

$$\delta_i$$
) $-\tilde{J}_l - \tilde{J}_t - \tilde{J}_b$] $-[-\bar{J}_l - \bar{J}_t - \bar{J}_b]$ }. (9)

Every classical CP scheme, including the unified CP scheme in Arakawa and Wu (2013), could be regarded as a special case in which $\left(\frac{\partial \bar{p}}{\partial t}\right)_{HYMACSS} = 0$ in

(9). Contrarily, HYMACSs omit this, so is more general. Still, the variables of the convective drafts and of the environment are to be diagnosed to close the equation set.

C. Closure of hybrid mass flux Kain-Fritsch scheme (HYMAKFS)

As a step to simple closure, following Kuell et al. (2007), the conventional assumption is used so that the variables of the environment are equal to the corresponding mean variables, thereby the tildes in (8) become over bars:

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{HYMACS} \cong \left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{HYMACS'} = \frac{1}{V} \left[\sum_{i} (\Psi_i + \varepsilon_i \overline{\psi} - \psi_i - \psi_i)\right]$$

$$\delta_i \psi_i - J_{it} \psi_{it} - J_{ib} \psi_{ib}) + \sum_i (-\varepsilon_i \bar{\psi} + \delta_i \psi_i)]. \tag{10}$$

Since convective time scale (10^3 s) is much shorter than synoptic one (10^5 s), after Kain and Fritsch (1993), convective drafts are assumed to be in a steady state so that the mass coupled ψ tendency in the convective drafts, the first summation in (10), turn zero, and this gives the closure of the conventional HYMACS (equation 12 in Kuell et al., 2007):

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{HYMACS'} \cong \left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{HYMAKFS} = \frac{1}{V} \sum_{i} (-\varepsilon_{i} \overline{\psi} + \delta_{i} \psi_{i}). \tag{11}$$

In comparison to Kuell et al. (2007), Kain and Fritsch (1993) used the classical assumption so that the net cumulus mass flux is zero, thereby \tilde{J} in (8) become $\bar{J} - \sum_i J_i$. Then, similarly, substituting $\tilde{\psi}$ with $\bar{\psi}$ and eliminating the first summation in (8) yield (equation 16.8 in Kain and Fritsch, 1993):

$$\left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{HYMACS'} \cong \left(\frac{\partial \overline{\rho \psi}}{\partial t}\right)_{KFSS} = \frac{1}{V} \sum_{i} (-\varepsilon_{i} \overline{\psi} + \delta_{i} \psi_{i} + J_{it} \psi_{it} + J_{ib} \psi_{ib}).$$
(12)

Last but not least, following KFSs, a one-dimensional entraining/detraining plume model is applied to diagnose the entrainment rate ε_i , the detrainment rate δ_i and the prognostic variables in convective drafts ψ_i in (11); i=u for the updraft plume, and i=d for the downdraft plume. Taking ψ as unity, potential temperature and specific moisture content in (11) yields the equation set of the HYMAKFS, in which the same trigger function and convective available potential energy removal process as the KFS in WRF are applied, but (11) is used instead of (12).

3. Ongoing projects and summary

9 idealized tropical cyclone simulations are designed to justify the hypotheses mentioned in section 1 as table 1. In table 1, the resolution of the model decreases from the left to the right, and the implicitness of cumulus convection decreases from the bottom to the top; "kf" simulations have implicit convective drafts and compensating motions; "hy" ones have implicit convective drafts but no implicit compensating motions; "no" ones have no implicit cumulus convection. The first hypothesis, implementing HYMACSs could be pivotal to the unified CP problem, would be verified by comparing the results between different horizontal grid spacing settings; as far as a scheme, the more the results converges, the more it is unified. The second hypothesis, applying HYMACSs might be a better way to simulate tropical cyclone intensification, would be confirmed by comparing the result of "no3" with that of the other simulations; "no3" is chosen as the control simulation because previous studies (e.g., Gentry and Lackmann, 2010) suggested it for operational simulations.

The model time step is the horizontal grid spacing divided by 200 m s⁻¹, which is short enough to satisfy Courant-Friedrichs-Lewy condition. The model domain is $5400 \text{ km} \times 5400 \text{ km} \times 25 \text{ km}$, which is large enough for internal gravity waves not to propagate back within a day, with doubly periodic lateral boundary condition and 21 vertical layers. All other settings follow the WRF default idealized tropical cyclone simulation

settings. For the physics, the Kessler, without-ice, microphysics scheme (Kessler, 1969), the Yonsei University planetary boundary layer scheme (Hong et al., 2006) and the Monin-Obukhov surface layer scheme (Paulson, 1970; Dyer and Hicks, 1970; Webb, 1970; Belijaars, 1994; Zhang and Anthes, 1982) are used, and the capped Newtonian relaxation scheme is used on potential temperature to approximate longwave radiation (Rotunno and Emanuel, 1987). For the environment, a constant Coriolis parameter of 5 × 10⁻⁵ s⁻¹ and a constant sea surface temperature of 28 °C are held. For the initial condition, a horizontally homogeneous motionless state is set by the mean hurricane season sounding for the West Indies area (Jordan, 1958), and

then a hydrostatic-and-gradient-wind balanced hurricane-like vortex with an outer radius of 412.5 km, a radius of max winds of 82.5 km, a max wind speed of 15 m s⁻¹ and a depth of 20 km (Rotunno and Emanuel, 1987) is placed in the middle of the domain.

To summarize, this study introduces hybrid mass flux cumulus schemes (HYMACSs), which solve convective drafts implicitly but compensating motions explicitly by a mass source/sink term in the continuity equation. It is hypothesized that the horizontally limited compensating subsidence in the KFS overheats a grid point, which further overestimates the intensity of a tropical cyclone. To verify the hypothesis, experiment is ongoing.

Table. 1. Simulations.

Horizontal grid spacing setting (km)	3	9	27
No cumulus parameterization scheme (no)	no3	no9	no27
Hybrid mass flux Kain-Fritsch scheme (hy)	hy3	hy9	hy27
Kain-Fritsch scheme (kf)	kf3	kf9	kf27

References

Anthes, R. A., 1977: A Cumulus Parameterization Scheme Utilizing a One-Dimensional Cloud Model. *Mon. Wea. Rev.*, **105**, 270-286.

Arakawa, A., and C.-M. Wu, 2013: A Unified Representation of Deep Moist Convection in Numerical Modeling of the Atmosphere. Part I. *J. Atmos. Sci.*, **70**, 1977-1992.

Beljaars, A.C.M., 1994: The parameterization of surface fluxes in large-scale models under free convection. *Quart. J. Roy. Meteor. Soc.*, **121**, 255-270.

Bjerknes, J., 1938: Saturated adiabatic ascent of air through dry adiabatically descending environment. *Quart. J. Roy. Meteor. Soc.*, **64**, 325-330.

Chandrasekar, R., and C. Balaji, 2012: Sensitivity of tropical cyclone Jal simulations to physics parameterizations. *J. Earth Syst. Sci.*, **121**, 923-946.

Dyer, A. J., and B. B. Hicks, 1970: Flux-gradient relationships in the constant flux layer. *Quart. J. Roy.*

Meteor. Soc., 96, 715-721.

Gentry, M. S., and G. M. Lackmann, 2010: Sensitivity of Simulated Tropical Cyclone Structure and Intensity to Horizontal Resolution. *Mon. Wea. Rev.*, **138**, 688-704.

Haghroosta, T., W. R. Ismail, P. Ghafarian, and S. M.
Barekati, 2014: The efficiency of the WRF model for simulating typhoons. *Nat. Hazards Earth Syst. Sci. Discuss.*, 2, 287-313.

Hong, S.-Y., Y. Noh, and J. Dudhia, 2006: A New Vertical Diffusion Package with an Explicit Treatment of Entrainment Processes. *Mon. Wea. Rev.*, **134**, 2318-2341.

Jordan, C. L., 1958: MEAN SOUNDINGS FOR THE WEST INDIES AREA. *J. Meteor.*, **15**, 91-97.

Kain, J. S., 2004: The Kain-Fritsch Convective Parameterization: An Update. *J. Appl. Meteor.*, **43**, 170-181.

Kain, J. S., and J. M. Fritsch, 1990: A One-Dimensional Entraining/Detraining Plume Model and Its Application

- in Convective Parameterization. *J. Atmos. Sci.*, **47**, 2784-2802.
- Kain, J. S., and J. M. Fritsch, 1993: Convective Parameterization for Mesoscale Models: The Kain-Fritsch Scheme. *Meteor. Monogr.*, **24**, 165-170.
- Kessler, E., 1969: On the distribution and continuity of water substance in atmospheric circulation. *Meteor. Monogr.*, 32, 84 pp.
- Kuell, V., A. Gassmann, and A. Bott, 2007: Towards a new hybrid cumulus parameterization scheme for use in non-hydrostatic weather prediction models. *Quart. J. Roy. Meteor. Soc.*, **133**, 479-490.
- Kuell, V., and A. Bott, 2008: A hybrid convection scheme for use in non-hydrostatic numerical weather prediction models. *Meteor. Z.*, 17, 775-783.
- Mandal, M., U. C. Mohanty, and S. Raman, 2004: A Study on the Impact of Parameterization of Physical Processes on Prediction of Tropical Cyclones over the Bay of Bengal with NCAR/PSU Mesoscale Model. *Nat. Hazards*, **31**, 391-414.
- Molinari, J., and M. Dudek, 1992: Parameterization of Convective Precipitation in Mesoscale Numerical Models: A Critical Review. *Mon. Wea. Rev.*, **120**, 326-344.
- Paulson, C. A., 1970: The Mathematical Representation of Wind Speed and Temperature Profiles in the Unstable Atmospheric Surface Layer. *J. Appl. Meteor.*, 9, 857-861.
- Prater, B. E., and J. L. Evans, 2002: Sensitivity of modeled tropical cyclone track and structure of hurricane Irene (1999) to the convective parameterization scheme. *Meteor. Atmos. Phys.*, **80**, 103-115.
- Rao, D. V. B., and D. H. Prasad, 2007: Sensitivity of tropical cyclone intensification to boundary layer and convective processes. *Nat. Hazards*, 41, 429-445.

- Rotunno, R., and K. A. Emanuel, 1987: An Air-Sea Interaction Theory for Tropical Cyclones. Part II: Evolutionary Study Using a Nonhydrostatic Axisymmetric Numerical Model. *J. Atmos. Sci.*, 44, 542-561.
- Singh, K. S., and M. Mandal, 2014: Sensitivity of Mesoscale Simulation of Aila Cyclone to the Parameterization of Physical Processes Using WRF Model. *Monitoring and Prediction of Tropical Cyclones in the Indian Ocean and Climate Change*, 300-308.
- Skamarock, W. C., and Coauthors, 2008: A description of the advanced research WRF version 3. NCAR Tech.

 Note NCAR/TN-475+STR, 113 pp.
- Srinivas, C. V., R. Venkatesan, D. V. B. Rao, and D. H. Prasad, 2007: Numerical Simulation of Andhra Severe Cyclone (2003): Model Sensitivity to the Boundary Layer and Convection Parameterization. *Pure appl. geophys.*, **164**, 1465-1487.
- Sun, Y., L. Yi, Z. Zhong, Y. Hu, Y. Ha, 2013: Dependence of model convergence on horizontal resolution and convective parameterization in simulations of a tropical cyclone at gray-zone resolutions. J. Geophys. Res., 118, 7715-7732.
- Webb, E. K., 1970: Profile relationships: the log-linear range, and extension to strong stability. *Quart. J. Roy. Meteor. Soc.*, **96**, 67-90.
- Yang, M.-J., and L. Ching, 2005: A Modeling Study of Typhoon Toraji (2001): Physical Parameterization Sensitivity and Topographic Effect. *T. A. O.*, **16**, 177-213.
- Zhang, D., and R.A. Anthes, 1982: A High-Resolution Model of the Planetary Boundary Layer—Sensitivity Tests and Comparisons with SESAME-79 Data. *J. Appl. Meteor.*, **21**, 1594-1609.