Developing Deep Atmospheric Dynamics for NCEP Global Forecast System

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Content

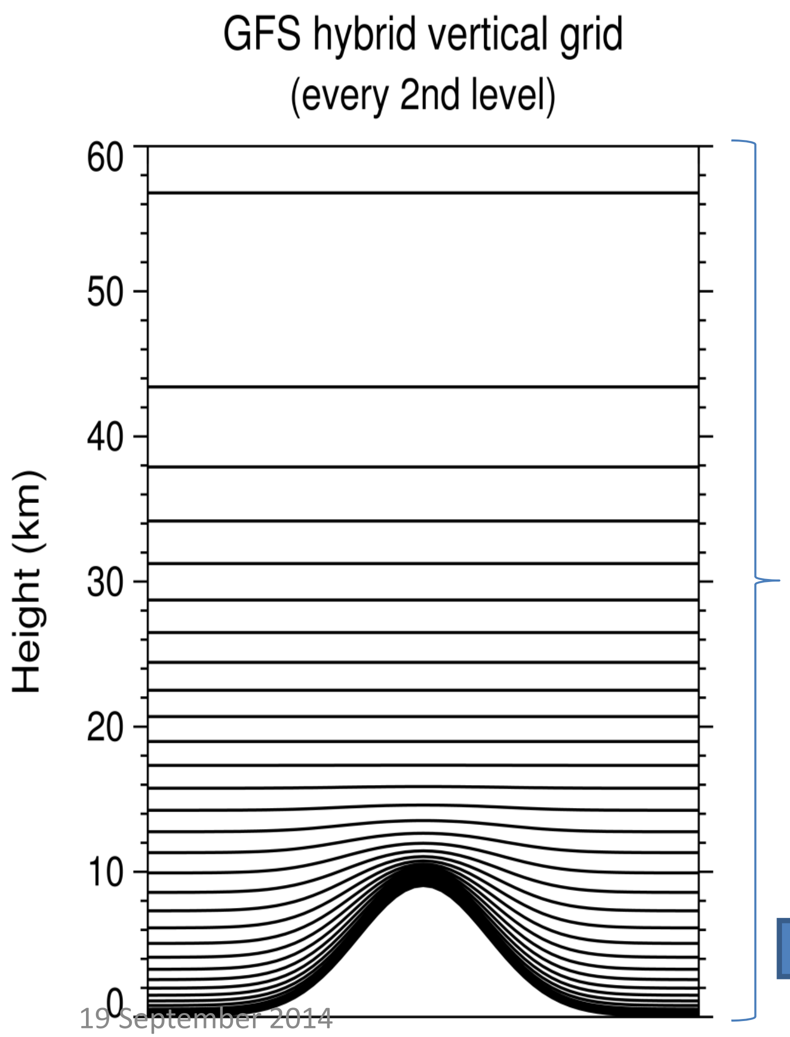
- To re-iterate the necessity for NCEP GFS to have deep atmospheric dynamics.
- To illustrate an alternated dynamics discretization to re-use most of NCEP GFS routines.

For NCEP GFS

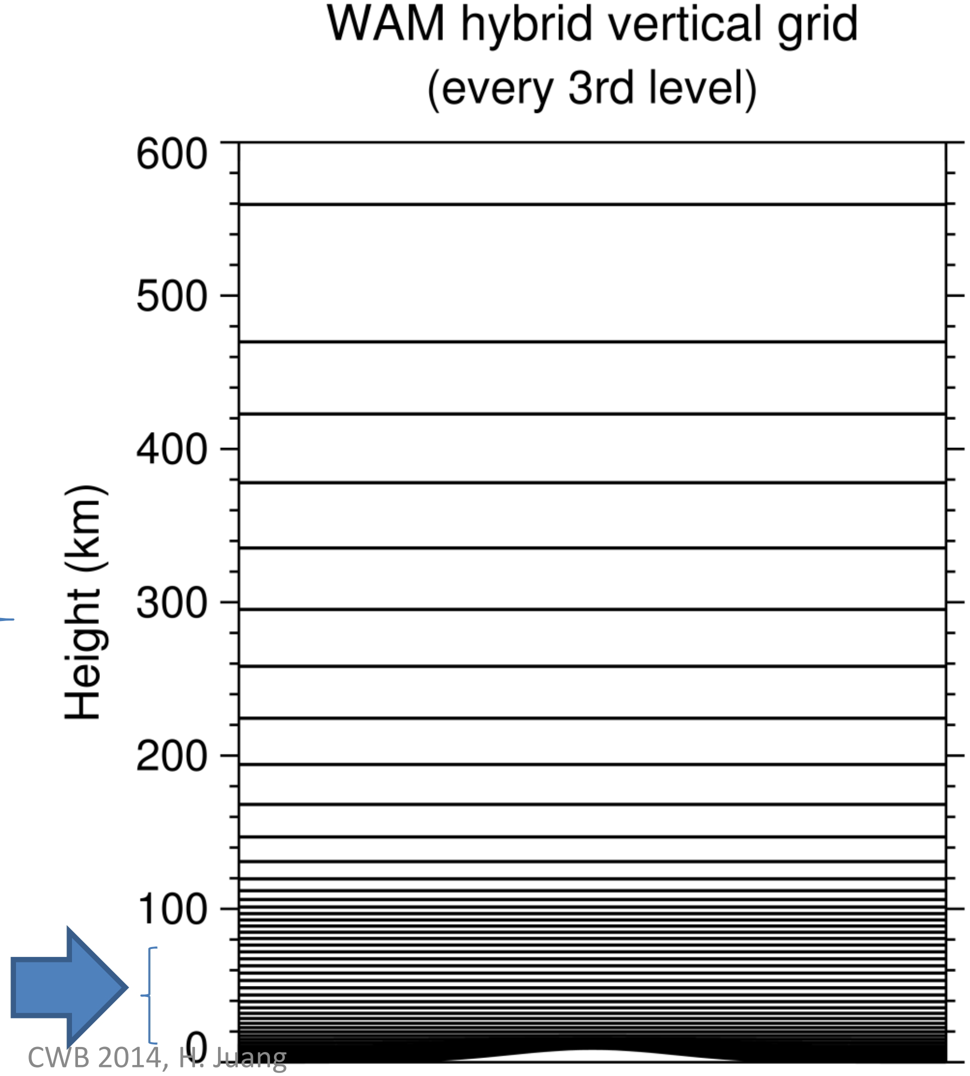
- NCEP requires GFS to extend its vertical domain to couple with space environmental model
- NCEP requires GFS to be nonhydrostatic ready for higher and higher resolution (NGGPS)
- We have done a hydrostatic system called WAM (whole atmosphere model) with taking care different gases for deep atmosphere (Juang 2011 MWR).
- We are implementing deep atmospheric dynamics for NCEP GFS, which is
  - Much more accurate than nonhydrostatic
  - Fully completing non-approximated dynamics

Opr GFS vs WAM

64 layers

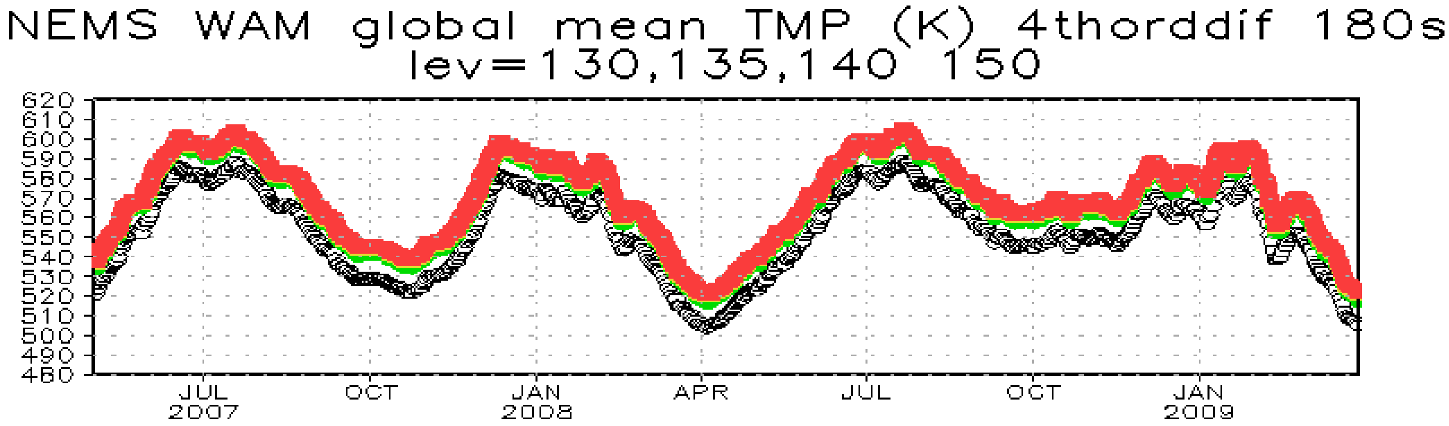


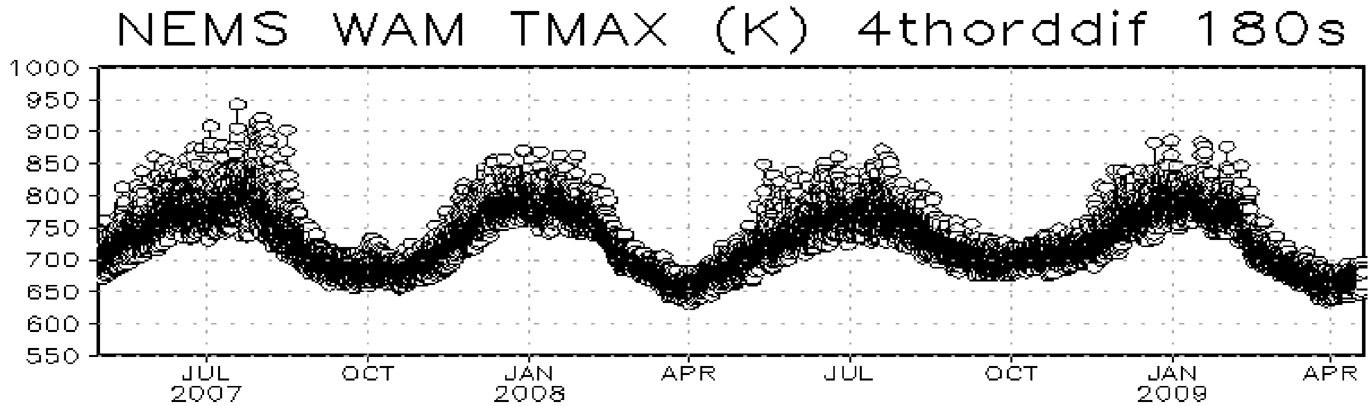
150 layers

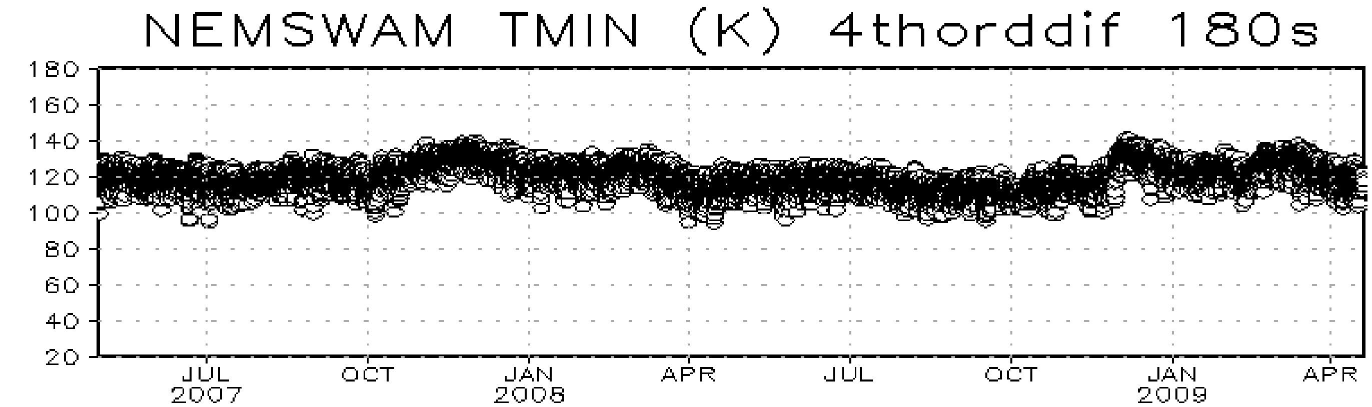


WAM Eulerian

dt=180sec

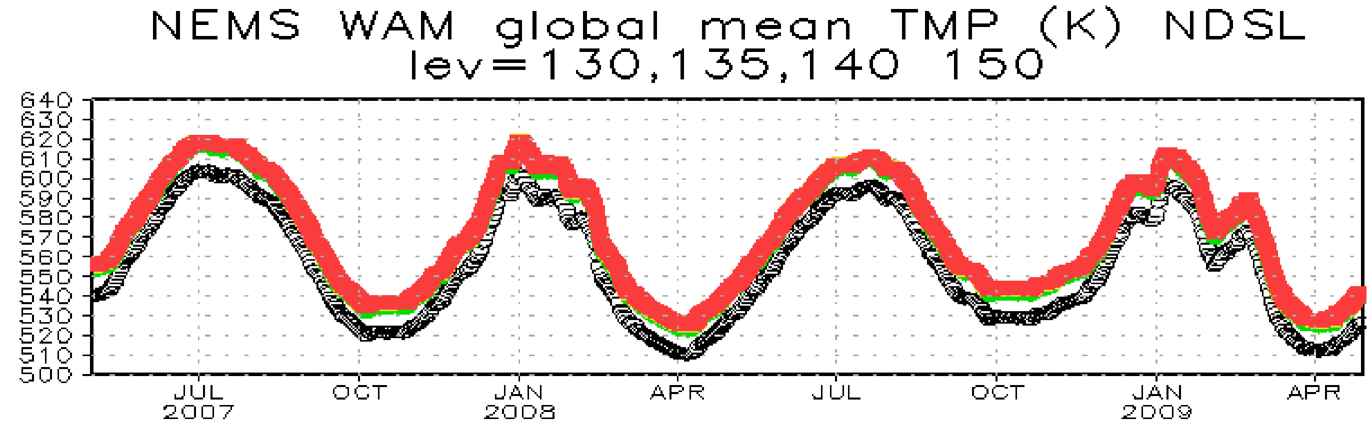


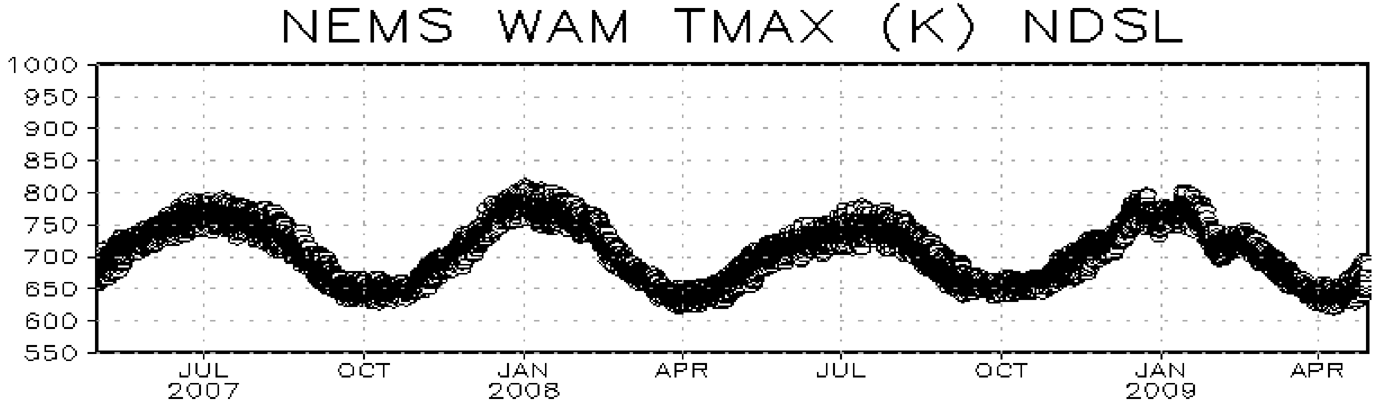


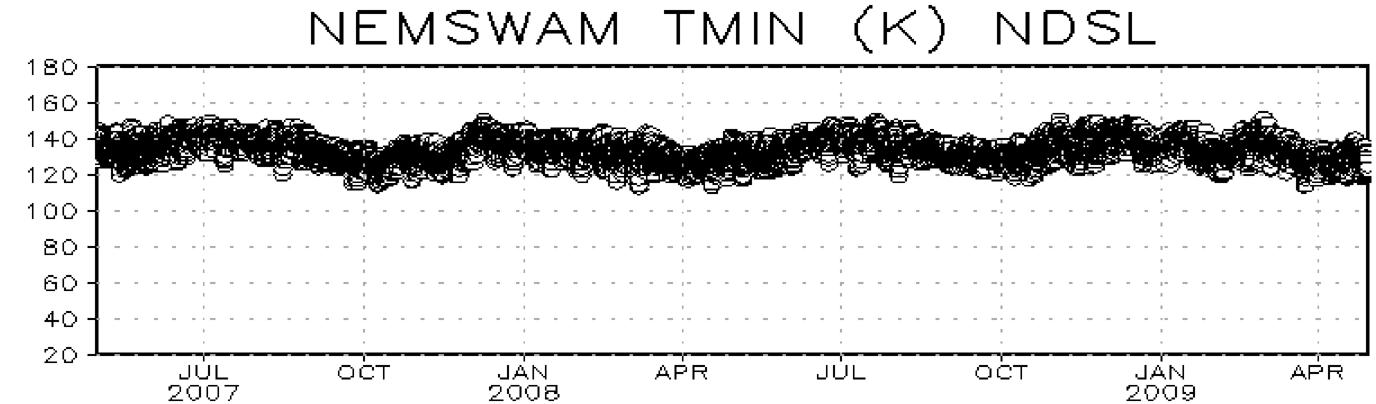


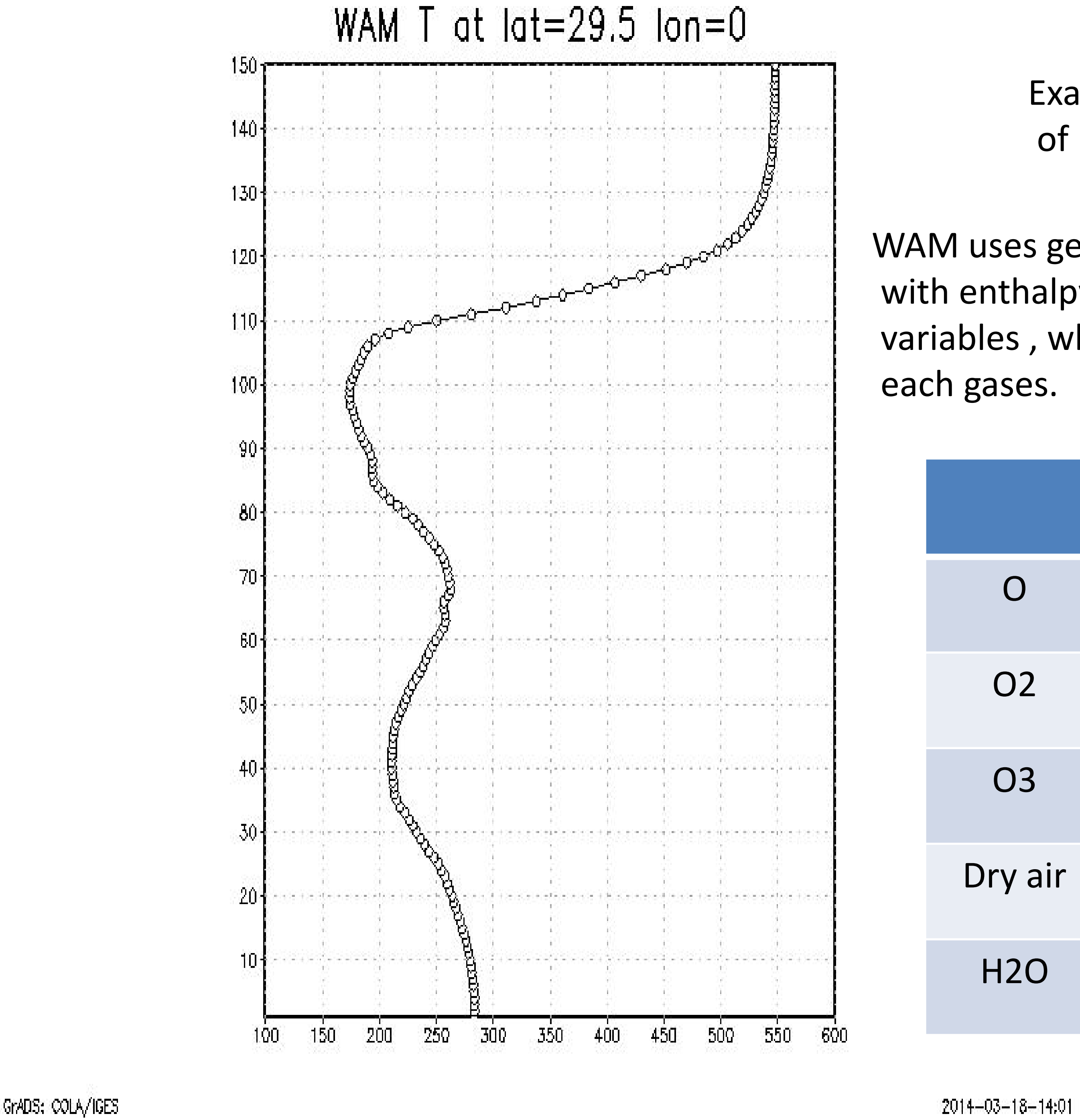
WAM NDSL

dt=400sec









Example of T profile
of 150 layers

WAM uses generalized hybrid coordinate with enthalpy CpT as thermodynamics variables , where Cp is summation of each gases.

	R	Cp
O	519.674	1299.18
O2	259.837	918.096
O3	173.225	820.239
Dry air	296.803	1039.64
H2O	461.50	1846.00

Maxima wind (m/s) at NCEP GFS 150 layers WAM

```
in do_dynamics_two_loop for spdmx at kdt=      40825
spdmx(001:010)=  19.  20.  21.  23.  25.  26.  27.  28.  28.  28.
spdmx(011:020)=  28.  27.  27.  27.  27.  28.  28.  28.  29.  30.
spdmx(021:030)=  31.  33.  35.  37.  40.  42.  44.  46.  49.  53.
spdmx(031:040)=  58.  61.  63.  63.  62.  60.  55.  47.  45.  44.
spdmx(041:050)=  45.  45.  47.  49.  52.  55.  59.  62.  65.  68.
spdmx(051:060)=  72.  76.  80.  84.  87.  90.  93.  95.  97.  98.
spdmx(061:070)= 102. 110. 118. 127. 135. 143. 149. 153. 155. 152.
spdmx(071:080)= 147. 145. 142. 138. 135. 132. 130. 126. 121. 119.
spdmx(081:090)= 114. 112. 110. 106. 100.  95.  94.  90.  89.  89.
spdmx(091:100)=  87.  82.  91.  95.  99.  97. 104. 100. 111. 120.
spdmx(101:110)= 125. 133. 148. 167. 172. 164. 159. 160. 147. 124.
spdmx(111:120)= 117. 125. 133. 138. 137. 157. 183. 202. 220. 243.
spdmx(121:130)= 269. 297. 319. 338. 355. 368. 378. 386. 392. 396.
spdmx(131:140)= 399. 402. 404. 405. 406. 407. 408. 409. 410. 410.
spdmx(141:150)= 411. 412. 412. 413. 413. 414. 414. 415. 415. 418.
```

Shallow (r=a) vs Deep (r=a+z)

- Assume at z=637.12km, so r = 1.1a
- For shallow dynamic

$$u = a\cos\phi \frac{d\lambda}{dt}$$

- For deep atmosphere

$$u = r\cos\phi \frac{d\lambda}{dt}$$

- For example, ϕ=45° , u=400m/s, after one hour advection, the displacement has about 1° error in λ.

Deep atmospheric equation in height & spherical coordinates

Momentum

$$\frac{du}{dt} - \frac{uv\tan\phi}{r} + \frac{uw}{r} - (2\Omega\sin\phi)v + (2\Omega\cos\phi)w + \frac{1}{\rho}\frac{\partial p}{r\cos\phi\partial\lambda} = F_u$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{r} + \frac{vw}{r} + (2\Omega\sin\phi)u + \frac{1}{\rho}\frac{\partial p}{r\partial\phi} = F_v$$

$$\frac{dw}{dt} - \frac{u^2+v^2}{r} - (2\Omega\cos\phi)u + \frac{1}{\rho}\frac{\partial p}{\partial r} + g = F_w$$

where

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + u\frac{\partial A}{r\cos\phi\partial\lambda} + v\frac{\partial A}{r\partial\phi} + w\frac{\partial A}{\partial r} \qquad r = a + z$$

$$u = r\cos\phi\frac{d\lambda}{dt}$$

$$v = r\frac{d\phi}{dt}$$

Gas law

$$p = \sum_n p_n = \left(\sum_n \rho_n R_n\right)T = \rho\left(\sum_n \frac{\rho_n R_n}{\rho}\right)T = \rho\left(\sum_n q_n R_n\right)T = \rho RT$$

Density

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{r\cos\phi\partial\lambda} + \frac{\partial\rho v\cos\phi}{r\cos\phi\partial\phi} + \frac{\partial\rho r^2 w}{r^2\partial r} = F_\rho$$

Deep Atmos vs non-Hydro

- From Deep atmosphere, we require r changes with time, thus we need dw/dt equation
- And we need full curvature and Coriolis force terms to satisfy conservation
- Thus, based on conservation requirement, **a deep atmospheric dynamic is a non-hydrostatic dynamic**. **A non-hydrostatic dynamics can be shallow or deep atmospheric dynamics**.
- Both r and vertical components of curvature and Coriolis force should be considered in deep atmosphere; and should not be considered in shallow atmosphere. (see Juang 2014 NCEP ON#477)

$$\frac{du^*}{dt}+\frac{u^*w}{r} \quad -f_s v^* \quad +f_c^* w \qquad +\frac{\kappa h}{p}\frac{1}{r}\left(\frac{\partial p}{\partial \lambda}-\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r}\frac{\partial r}{\partial \lambda}\right) \quad =F_u$$

$$\frac{dv^*}{dt}+\frac{v^*w}{r} \quad +f_s u^* \quad +m^2\frac{s^{*2}}{r}\sin\phi \quad +\frac{\kappa h}{p}\frac{1}{r}\left(\frac{\partial p}{\partial \varphi}-\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r}\frac{\partial r}{\partial \varphi}\right) \quad =F_v$$

$$\frac{dw}{dt}-m^2\frac{s^{*2}}{r} \qquad -m^2 f_c^* u^* \qquad +\frac{\kappa h}{p}\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r} \qquad +g \;=F_w$$

$$\frac{dh}{dt}-\frac{\kappa h}{p}\frac{dp}{dt}=F_h$$

Deep Atmos Dyn
in spherical mapping
& generalized coordinates

$$\frac{\partial \rho^*}{\partial t}+m^2\frac{\partial \rho^*\frac{u^*}{r}}{\partial \lambda}+m^2\frac{\partial \rho^*\frac{v^*}{r}}{\partial \varphi}+\frac{\partial \rho^*\dot{\zeta}}{\partial \zeta}=F_\rho^*$$

Staniforth and Wood (2003)
Juang (2014) NCEP Office Note

$$\frac{dq_i}{dt}=F_{q_i}$$

$$p=\rho\kappa h$$

where $$\frac{d()}{dt}=\frac{\partial()}{\partial t}+\dot{\lambda}\frac{\partial()}{\partial \lambda}+\dot{\varphi}\frac{\partial()}{\partial \varphi}+\dot{\zeta}\frac{\partial()}{\partial \zeta}=\frac{\partial()}{\partial t}+m^2u^*\frac{\partial()}{r\partial \lambda}+m^2v^*\frac{\partial()}{r\partial \varphi}+\dot{\zeta}\frac{\partial()}{\partial \zeta} \quad ; \quad \rho^*=\rho\frac{r^2}{a^2}\frac{\partial r}{\partial \zeta}$$

$$f_s=2\Omega\sin\phi \;;\; f_c^*=2\Omega\cos^2\phi \;;\; g=g(r) \;;\; \kappa=\frac{R}{C_P} \;;\; \gamma=\frac{C_P}{C_V} \;;\; s^{*2}=u^{*2}+v^{*2}$$

In hydrostatic system, (shallow atmosphere)
Continuity equation in generalized coordinate is written as

$$\frac{\partial \frac{\partial p}{\partial \zeta}}{\partial t} + m^2 \left(\frac{\partial \frac{\partial p}{\partial \zeta} u^*}{a \partial \lambda} + \frac{\partial \frac{\partial p}{\partial \zeta} v^*}{a \partial \varphi} \right) + \frac{\partial \frac{\partial p}{\partial \zeta} \dot{\zeta}}{\partial \zeta} = 0$$

In deep atmosphere, continuity equation is

$$\frac{\partial \rho^*}{\partial t} + m^2 \frac{\partial \rho^* \frac{u^*}{r}}{\partial \lambda} + m^2 \frac{\partial \rho^* \frac{v^*}{r}}{\partial \varphi} + \frac{\partial \rho^* \dot{\zeta}}{\partial \zeta} = 0$$

We want to make it the same form to use original routines.

To derive deep-atmosphere continuity equation into the same form as shallow atmosphere

$$\frac{\partial \rho^*}{\partial t} + m^2 \frac{\partial \rho^* \frac{u^*}{r}}{\partial \lambda} + m^2 \frac{\partial \rho^* \frac{v^*}{r}}{\partial \varphi} + \frac{\partial \rho^* \dot{\zeta}}{\partial \zeta} = 0$$

We define a coordinate pressure as hydrostatic one based on the coordinate density as

$$\frac{\partial \tilde{\bar{p}}}{\partial \zeta} = -\rho^* g_0$$ where g_0 is constant

Put it into deep-atmosphere continuity equation

We have

$$\frac{\partial \frac{\partial \tilde{\bar{p}}}{\partial \zeta}}{\partial t} + m^2 \left(\frac{\partial \frac{\partial \tilde{\bar{p}}}{\partial \zeta} \frac{u^*}{r}}{\partial \lambda} + \frac{\partial \frac{\partial \tilde{\bar{p}}}{\partial \zeta} \frac{v^*}{r}}{\partial \varphi} \right) + \frac{\partial \frac{\partial \tilde{\bar{p}}}{\partial \zeta} \dot{\zeta}}{\partial \zeta} = 0$$

In deep atmosphere, horizontal wind with Gaussian weighting

$$u^* = r\cos^2\phi\frac{d\lambda}{dt} = u\cos\phi \qquad v^* = r\cos\phi\frac{d\phi}{dt} = v\cos\phi$$

We can define, height-weighted horizontal wind as

$$\tilde{u} = a\cos^2\phi\frac{d\lambda}{dt} = \frac{a}{r}u^* \qquad \tilde{v} = a\cos\phi\frac{d\phi}{dt} = \frac{a}{r}v^*$$

Modify continuity equation

$$\frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}}{\partial t} + m^2\left(\frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}\frac{u^*}{r}}{\partial\lambda} + \frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}\frac{v^*}{r}}{\partial\varphi}\right) + \frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}\dot{\zeta}}{\partial\zeta} = 0$$

We have

$$\frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}}{\partial t} + m^2\left(\frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}\tilde{u}}{a\partial\lambda} + \frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}\tilde{v}}{a\partial\varphi}\right) + \frac{\partial\frac{\partial\tilde{\tilde{p}}}{\partial\zeta}\dot{\zeta}}{\partial\zeta} = 0$$

Since $\frac{\partial \tilde{\bar{p}}}{\partial \zeta} = -\rho^* g_0$ and $\rho^* > 0$

Thus $\tilde{\bar{p}}$ is monotone with vertical coordinate

We can use it for coordinate definition

For opr compatibility, we use $$\hat{\tilde{\bar{p}}}_k = \hat{A}_k + \hat{B}_k \tilde{\bar{p}}_s$$

and relation with height as

$$\frac{\partial \tilde{\bar{p}}}{\partial \zeta} = -\rho g_0 \frac{r^2}{a^2} \frac{\partial r}{\partial \zeta} \qquad \text{because} \qquad \rho^* = \rho \frac{r^2}{a^2} \frac{\partial r}{\partial \zeta}$$

So we are using height-weighted horizontal wind, and let $\varepsilon = \dfrac{r}{a}$

$$\tilde{u} = \frac{a}{r} u^* = \frac{u \cos\phi}{\varepsilon} \qquad \tilde{v} = \frac{a}{r} v^* = \frac{v \cos\phi}{\varepsilon}$$

all places having wind should be modified

$$\frac{d()}{dt} = \frac{\partial()}{\partial t} + m^2 u^* \frac{\partial()}{r\partial\lambda} + m^2 v^* \frac{\partial()}{r\partial\varphi} + \dot{\zeta}\frac{\partial()}{\partial\zeta} = \frac{\partial()}{\partial t} + m^2 \tilde{u} \frac{\partial()}{a\partial\lambda} + m^2 \tilde{v} \frac{\partial()}{a\partial\varphi} + \dot{\zeta}\frac{\partial()}{\partial\zeta}$$

$$\frac{d\tilde{u}}{dt} = \frac{a}{r}\frac{du^*}{dt} - \frac{au^*}{r^2}\frac{dr}{dt} = -2\tilde{u}\frac{w}{\varepsilon a} + f_s \tilde{v} - f_c^* \frac{w}{\varepsilon} - \frac{\kappa h}{p\varepsilon^2}\frac{\partial p}{a\partial\lambda} - \frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial\tilde{\bar{p}}}\frac{g_0 \partial r}{a\partial\lambda}$$

$$\frac{d\tilde{v}}{dt} = \frac{a}{r}\frac{dv^*}{dt} - \frac{av^*}{r^2}\frac{dr}{dt} = -2\tilde{v}\frac{w}{\varepsilon a} - f_s \tilde{u} - m^2 \frac{\tilde{s}^2}{a}\sin\phi - \frac{\kappa h}{p\varepsilon^2}\frac{\partial p}{a\partial\varphi} - \frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial\tilde{\bar{p}}}\frac{g_0 \partial r}{a\partial\varphi}$$

From angular momentum principle, we have relation between model layer and model level as

$$r_k^3 = \frac{1}{2}\hat{r}_{k+1}^3 + \frac{1}{2}\hat{r}_k^3$$

so

$$r_k^2 w_k = \frac{1}{2}\hat{r}_{k+1}^2 \hat{w}_{k+1} + \frac{1}{2}\hat{r}_k^2 \hat{w}_k$$

let

$$\tilde{w} = \frac{r^2}{a^2} w$$

we have

$$\tilde{w}_k = \frac{1}{2}\hat{\tilde{w}}_{k+1} + \frac{1}{2}\hat{\tilde{w}}_k$$

so vertical momentum eq can be written as

$$\frac{d\tilde{w}}{dt} = \frac{2r}{a^2} w^2 + \frac{r^2}{a^2}\frac{dw}{dt} = 2\frac{\tilde{w}^2}{\varepsilon^3 a} + m^2\varepsilon^3 \frac{\tilde{s}^2}{a} + m^2\varepsilon^3 f_c^* \tilde{u} + g_0\left(\varepsilon^4 \frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial \tilde{\bar{p}}} - 1\right)$$

with

$$g = \frac{a^2}{r^2} g_0$$

In short summary, we have following

$$\frac{d\tilde{u}}{dt}+2\frac{\tilde{u}\tilde{w}}{\varepsilon^3 a}-f_s\tilde{v}+f_c^*\frac{\tilde{w}}{\varepsilon^3}+\frac{\kappa h}{p\varepsilon^2}\frac{\partial p}{a\partial\lambda}-+\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial\tilde{\bar{p}}}\frac{g_0\partial r}{a\partial\lambda}=0$$

$$\frac{d\tilde{v}}{dt}+2\frac{\tilde{v}\tilde{w}}{\varepsilon^3 a}+f_s\tilde{u}+m^2\frac{\tilde{s}^2}{a}\sin\phi+\frac{\kappa h}{p\varepsilon^2}\frac{\partial p}{a\partial\varphi}+\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial\tilde{\bar{p}}}\frac{g_0\partial r}{a\partial\varphi}=0$$

$$\frac{d\tilde{w}}{dt}-2\frac{\tilde{w}^2}{\varepsilon^3 a}-m^2\varepsilon^3\frac{\tilde{s}^2}{a}-m^2\varepsilon^3 f_c^*\tilde{u}-g_0\left(\varepsilon^4\frac{\partial p}{\partial\zeta}\frac{\partial\zeta}{\partial\tilde{\bar{p}}}-1\right)=0$$

$$\frac{\partial\frac{\partial\tilde{\bar{p}}}{\partial\zeta}}{\partial t}+m^2\left(\frac{\partial\frac{\partial\tilde{\bar{p}}}{\partial\zeta}\tilde{u}}{a\partial\lambda}+\frac{\partial\frac{\partial\tilde{\bar{p}}}{\partial\zeta}\tilde{v}}{a\partial\varphi}\right)+\frac{\partial\frac{\partial\tilde{\bar{p}}}{\partial\zeta}\dot{\zeta}}{\partial\zeta}=0$$

So prognostic variables are $\tilde{u}\quad\tilde{v}\quad\tilde{w}\quad\tilde{\bar{p}}\quad h\quad p$

In rest deep atmosphere, hydrostatic equilibrium should be

$$\varepsilon^4 \frac{\partial \bar{p}}{\partial \zeta} \frac{\partial \zeta}{\partial \tilde{\bar{p}}} - 1 = 0$$

So in discretization form, we have $\frac{\Delta \tilde{\bar{p}}}{\Delta \bar{p}} = \varepsilon^4$ (1)

To have hydrostatic pressure, we may need iteration by coordinate pressure as

$$\frac{\partial \tilde{\bar{p}}}{\partial \zeta} = -\rho g_0 \frac{r^2}{a^2} \frac{\partial r}{\partial \zeta} = -\frac{p g_0}{\kappa h} \frac{a}{3} \frac{\partial \varepsilon^3}{\partial \zeta} = -\frac{\left(\bar{p} + p'\right) g_0}{\kappa h} \frac{a}{3} \frac{\partial \varepsilon^3}{\partial \zeta} \qquad (2)$$

Given coordinate pressure, perturbed pressure, and temperature, with guess hydrostatic pressure, we can get height factor by (2), then use height factor to get hydrostatic pressure by (1), then back to (2) and so on. Solve it by entire given column together.

Summary

- Deep atmospheric dynamics is discretized in generalized hybrid coordinates with possible to reuse existed routines.
- A coordinate pressure is introduced for both continuity equation and height transformation.
- Height-weighted horizontal wind is used in the code so continuity equation has the same form as shallow atmosphere.
- Height-weighted vertical wind is used due to the relation between wind at levels and layers.
- Deep-atmosphere hydrostatic state requires gravitation force is function of height.
- Model is recoded with height-weighted wind and to add hydrostatic base to be more accurate in computing w equation.