Numerical modeling of the potential threat in our coastal environment: Generation of freak waves

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Abstract

The so-called freak waves are exceptionally large, steep, and asymmetric waves whose heights usually exceed by 2.2 times the significant wave height. These waves, described as 'holes in the sea' or 'wall of waters', have been long known to be notorious hazards to navigation vessels and marine structures. With little warning, these transient giant and steep waves can mysteriously occur from deep-water wave groups in random open seas. Many freak waves' devastating impacts and sinister marine episodes have raised great interests in predicting their occurrence. Over the past two decades a great deal of efforts has been paid to examine the mechanisms that cause formation of freak waves. In this study, the purpose is to reproduce the potential threat in our coastal environment - generation of freak waves in a numerical wave tank. A higher-order non-hydrostatic model in a σ-coordinate system was developed. The model used an implicit finite difference scheme on a staggered grid to solve the unsteady Navier-Stokes equations with the free-surface boundary conditions simultaneously. Besides, an integral method was employed to resolve the top-layer non-hydrostatic pressure, allowing for accurately resolving free-surface wave propagation. Model accuracy was validated by linear/nonlinear progressive waves and nonlinear bi-chromatic deep-water wave groups. The model was then used to examine the two-dimensional and three-dimensional freak waves. Features of downshifting focusing location and wave asymmetry characteristics are predicted on the temporal and spatial domains of a freak wave. In the near future, an effective freak wave warming system could be developed by the present modeling framework together with sufficient field observation data.

Keywords: σ-coordinate; Non-hydrostatic pressure; Free-surface waves; Freak waves

I. Introduction

Wave grouping is a prominent feature in the ocean. Wave-wave interactions resulting from non-linear coupling of wave components can cause a great shift of energy re-distribution, significantly influencing the evolution of deep-water wave groups. In the presence of near-resonant interactions, modulation of wave components occur and the wave groups thereafter become unstable. For instance, the shape of an initial wave group can evolve into massive wave pulses while a single carrier wave travels with a pair of side-bands. Similarly, bi-chromatic waves that consist of two waves with a small period difference can also evolve several interesting physical characteristics such as the asymmetric wave profile, the convergence of wave energy, the modulation (demodulation) processes, and the recurrence of initial state (Hwung and Chiang, 2005). In contrast to slowly modulated wave groups mentioned above, fast modulated waves appear in spatial-temporal focusing of wave-wave and wave-current interactions (Baldock et al., 1996; Wu and Nepf, 2002; Wu and Yao, 2004). Owing to strong non-linearity in the modulated wave groups, the limiting focused wave crest is much larger than that predicted by linear wave theory (Baldock et al., 1996). Indeed, recently studies suggest that both slowly and fast modulated deep-water wave groups play an important role in the generation of freak waves that have cause numerous sinister marine episodes (Kharif and Pelinovsky, 2003). As a consequence, accurate prediction of the nature of non-linear deep-water wave groups is essential.

The purpose of this paper is to reproduce the potential threat in our coastal environment - generation of freak waves in a numerical wave tank. An efficient and accurate non-hydrostatic model capable of simulating strongly modulated deep-water wave groups was developed. Model accuracy was validated by linear/nonlinear progressive waves and nonlinear bi-chromatic deep-water wave groups. The model was then used to examine the two-dimensional and three-dimensional freak waves. Model results regarding the evolution of non-linear deep-water wave groups are discussed.

II. Non-hydrostatic Model

The non-hydrostatic model (Young & Wu, 2010) solves the unsteady, incompressible, Navier-Stokes equations to simulate free surface wave motions based on the staggered finite difference Crank-Nicholson scheme in the transformed σ domain. The implicit numerical algorithm relaxes the stability restriction of computational time steps in comparison to the explicit ones. Besides, the algorithm treats a 3D problem as a series of 2D vertical planes to yield block (hepta) diagonal system matrixes that can be solved directly without iterations. The boundary-fitted co-ordinate ensures an accurate representation of the free surface elevation and the irregular bottom topography. Most importantly, the top-layer pressure treatment using a cubic polynomial interpolation can provide further accuracy for phase velocity. The developed NHS model has been carefully validated against either analytical solutions or experimental data for various wave problems from shallow to deep water depth (Young et al., 2009; Young & Wu, 2010), clearly demonstrating the model's efficiency (requirement of only 2~5 vertical layers) and accuracy (dispersive degree up to Kh = 3~15).

III. Validation

In this section we aim to examine the present model’s accuracy in simulating linear dispersion, wave non-linearity, and wave-wave interaction. We use a 2D vertical numerical wave tank with a length of 10 times wavelength and a still water depth 1 *m*. The sponge layer is used to absorb outgoing waves and minimize wave reflection. In the model, 40 cells per wave length and five stretched vertical layers are used to discretize the computational domain. The time step is determined by setting the Courant number *Cr* = 0.5, where *c* is the wave speed.

1. Linear Dispersion

Frequency dispersion is an essential property for water wave propagation. To examine the present model’s capability in predicting linear wave dispersion, three non-dimensional relatively deep-water wave conditions, i.e. $Kh = \pi$, 3π, and 5π, are considered. An infinitesimal incident wave amplitude $a/h = 0.001$ is used to ensure linear wave condition.

Fig. 1 compares the steady spatial profiles of free-surface displacement predicted by the present model, Young et al.’s model (2007), and Yuan and Wu’s model (2006), with the analytical solutions. It can be seen that all three models accurately predicts wave propagation for $Kh = \pi$. As the dimensionless relative water depth increases, the present model is capable resolve linear dispersion in the extremely deep-water condition (i.e., $Kh = 5\pi$) while the other two models under-estimates wave length.

2. Wave Non-linearity

To examine the present model’s capability of simulating wave non-linearity, three wave steepness ranging from weak to strong non-linearity, i.e. $aK = \varepsilon = 0.10$, 0.20, and 0.30, of progressive Stokes waves at $Kh = \pi$ are considered.

Fig. 2 shows the excellent comparison of free-surface displacement time series at $x=3\lambda$ between the model results and analytical solutions based upon the fifth-order Stokes theory (Fenton, 1985). The present model faithfully predict vertically asymmetric wave profiles with a higher and narrower crest as well as a wider and less deep trough as wave steepness aK increases. Overall, both wave speed and amplitude are well resolved.

3. Wave-wave Interaction

The model is further validated against the bi-chromatic wave experiments of Hwung and Chiang (2005). We choose the different non-breaking experimental condition B11. The numerical tank is 300 *m* long with a constant water depth of 3.5 *m*. A horizontal grid spacing $\Delta x = 0.2$ *m* and five stretched vertical layers are used to discretize the computational domain. At the last 50 *m* of the tank, the sponge layer technique is applied to minimize wave reflection. Courant number Cr=0.1 is used to determine the time step. The total simulation time is 400 *s*.

Fig. 3 compares the predicted wave

profiles. The initial wave trains at $K_c x$ = 23 seem to be a linear superposition of two incident wave components. As the wave trains travel over a short distance, i.e. at $K_c x$ = 58, the envelope has steep fronts and gently sloping rears, consistent with several other experimental results. In the consecutive stage $K_c x$ = 114, the combination effects of frequency dispersion and non-linear effect result in energy focusing into the center of the wave group. Afterward, the de-modulation process of wave trains can be observed at $K_c x$ = 161, indicating a recurrence of initial stage. Until the end of the tank, the processes of modulation and de-modulation would cyclically repeat. The MLB model has a good phase agreement but under-estimates the wave amplitude, similar to those results in case B39. The FNLS model predicts better amplitude but fails to capture the phase. The patterns of predicted free-surface displacements at several stations, e.g. $K_c x$ = 58, 246, and 331, are totally different from those of the experimental data. In contrast to MLB and FNLS models (Chiang et al., 2007), the present non-hydrostatic model faithfully captures both the phase and wave amplitude. Overall, with the capability of resolving non-linearity and dispersion, the higher-order non-hydrostatic model clearly simulates the features of bi-chromatic wave trains.

IV. Applications for Freak waves

a numerical tank of 25m long with an undisturbed mean water depth of h = 0.6m is used. At the end of the tank, a combination of a 5m sponge layer technique and a radiation is applied to minimize the wave reflection. At the inflow boundary condition, a deep-water spectral wave packet with 32 components ranging from 0.6855 to 1.4745 s^{-1} is considered, based upon the experimental setup by Wu and Nepf (2002). In the model we use a spatial-temporal focusing method (Wu and Yao, 2004) to generate a freak wave.

The predicted spatial profile of a 2D freak wave is demonstrated in Fig. 4(a), indicating the feasibility of the dispersive spatial-temporal focusing mechanism in forming a freak wave. A detailed velocity field of a freak wave shows a dramatic change of velocity near the focusing location, which could exert tremendous hydrodynamic force on structures. Fig. 4(b) compares the predicted time series of surface displacements and the linear solution by super positioning the wave components with the measured data by Wu and Nepf (2002). It can be found that the wave crest and trough by the linear solution is under-predicted 20% and over-predicted 25%, respectively. Similar results are also reported by Baldock et al. (1996), who reported the important nonlinear wave-wave interactions in the case of a focusing wave train. On the other hand, the simulated wave profile is in excellent agreement with the experimental data. In addition, the asymmetry between the wave crest and the wave trough can be described by a crest-and-trough asymmetry factor, i.e. $\eta_c/(\eta_c - \eta_t)$ = 0.69, featuring a ‘‘wall of water’’ of a freak wave.

Next, we further apply the model to predict a 3D spatially focusing freak wave. Figure 5(a) shows the perspective view of the wave field in the vicinity of the focusing event. Prior to the freak wave, that is $t*=-1$ to −0.5s, one can clearly see a ‘hole in the sea’ that features an exceptionally large, steep, and asymmetric wave, that is the ‘wall of waters’. Consecutively, an extreme wave, freak wave, is rapidly developed at $t*$=0s with the amplitude that is larger than 4 or 5 times of the average wave height of the wave packet, which is the feature of freak waves. Shortly, the freak wave disappears in a second ($t*$=1s), consistent with the short-live feature documented by field observations. Figure 5(b) shows that the model results of the time series free-surface elevation at x = x_f =3.3m are in good agreement with experimental data. In particular, the model well predicts the surface displacements at different lateral locations, that is y=0, 0.3, 0.6, 0.9, and 1.5m of the wave crest. The water surface exhibits a distinctive crescent shape.

V. Summary and Conclusions

A σ-coordinate non-hydrostatic model with a new higher-order top-layer pressure treatment was developed. Accuracy of the model is carefully examined. It is shown that the model using only five vertical layers is capable of resolving wave non-linearity up to $aK = 0.30$ and linear dispersion up to $Kh = 15$.

For the bi-chromatic waves, stronger modulation and de-modulation processes occur due to larger wave steepness and smaller period difference. In contrast to MLB and FNLS models (Chiang et al., 2007), the non-hydrostatic model accurately predicts both wave amplitude and phase of the bi-chromatic waves. For the 2D/3D focusing freak waves, similarly, larger wave steepness and narrower band-width lead to stronger non-linear wave-wave interactions. The transient behavior and energy transfer of focusing waves are faithfully represented by the model. The model results and the experimental data are in very good agreement.. Results will be reported later.

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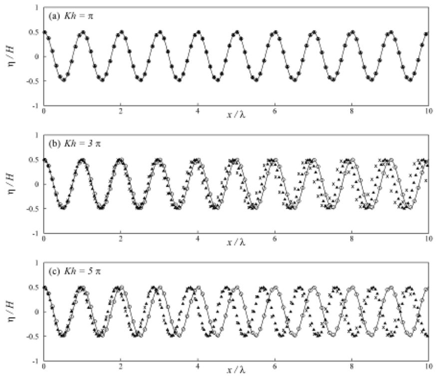


Fig. 1. Comparison of the predicted spatial free-surface displacement for (a) $Kh = \pi$, (b) 3 π, and (c) 5 π: analytical solution (solid lines), Yuan and Wu's model (X marks), Young et al.'s model (solid triangles), and the present model (open circles).

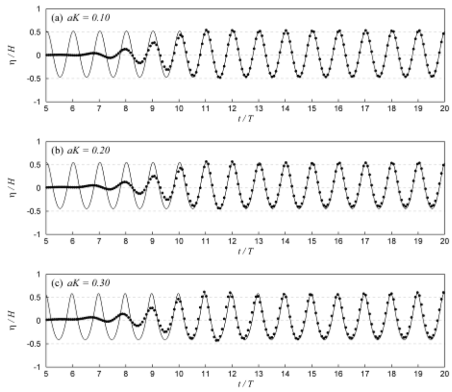


Fig. 2. Comparison of the predicted free-surface displacement time series at x = 3λ for (a) aK= 0.10, (b) aK = 0.20, and (c) aK = 0.30: analytical solutions (solid lines) and model results (solid circles).

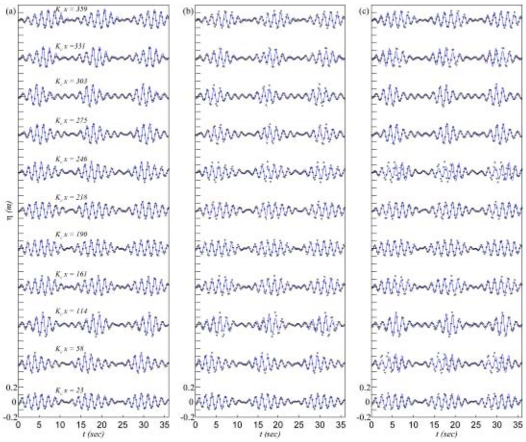
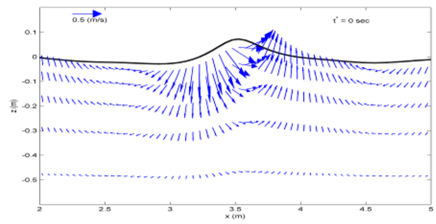


Fig. 3. Comparison of the predicted free-surface displacement time series for case B11 among (a) the present model, (b) the MLB model, and (c) the FNLS model: experimental data (open circles) and model results (solid lines).

(a)



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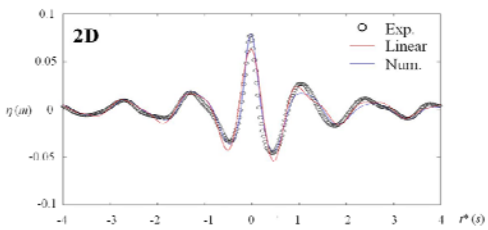
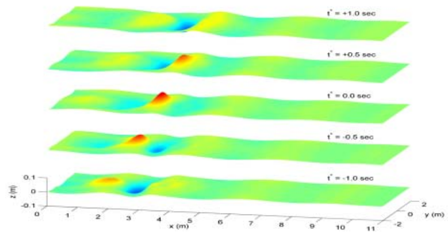


Fig. 4. (a) The spatial free surface profile and velocity field at the focusing location and (b) comparison of the surface displacement time series at the focusing location by the model (solid line), linear solution (dashed line), and measured data (circles).

(a)



(b)

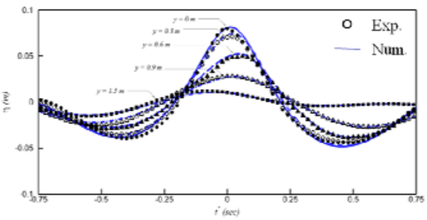


Fig. 5. (a) A perspective view for the evolution of a 3D spatially focusing freak wave and (b) Comparison of the predicted free-surface displacement time series along lateral direction (i.e. y=0, 0.3, 0.6, 0.9, and 1.5 m) for 3D spatially focusing freak waves: experimental data (symbols) and five-layer model (lines)