Developing Deep Atmospheric Dynamics for NCEP GFS

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Abstract

Deep atmospheric dynamic system is introduced and implemented into NCEP-GFS (National Centers for Environmental Prediction-Global Forecast System). The deep atmospheric system has no approximation and covers nonhydrostatic system with three dimensional momentum equations, three-dimensional Coriolis force, vertical varied gravitational force, and height varied momentum. A form of the discretized equation set to use the existed hydrostatic GFS routines without altering too much is illustrated.

Key word: deep atmosphere, nonhydrostatic system, global spectral model

1. Introduction

The trend of recent global modeling development is to move from hydrostatic system to nonhydrostatic system, however, in EMC (Environmental Modeling Center), for operational purposes and supporting all centers' missions in NCEP (National Centers for Environmental Prediction), we are not only having nonhydrostatic system for all centers' weather and climate forecasting but also working to have capability to support SWPC (Space Weather Prediction Center). Thus, we are not only in the stage to prepare nonhydrostatic cloud-resolvable for high-resolution weather/climate forecast but also in the requirement to have deep-atmospheric capabilities in order to couple with space weather model in NCEP GFS (Global Forecast System). However, deep-atmospheric dynamic modeling is not new, about a decade ago, UK Meteo Office is planning to do it as Staniforth and Wood (2003) and Wood and Staniforth (2003). Since deep atmospheric dynamic includes nonhydrostatic system, deep atmospheric dynamic system is our modeling goal.

For deep-atmospheric dynamics, the Euler equation set has capability to cover all scales of atmospheric motion. Based on conservative principles; such as angular momentum principle, energy conservation, mass conservation and entropy conservation, the deep atmospheric Euler equation set has no approximation and covers nonhydrostatic system with three dimensional momentum equations, three-dimensional Coriolis force, vertical varied gravitational force, and height varied momentum. For precise/correct deep atmospheric circulation to couple with space weather and prolong its predictability, this set of equation has been discretized into differencing equation set to be used for numerical modeling, which can be found in Juang (2014). An alternative form of this discretized equation set to use existed GFS routines without altering too much is presented in this extended abstract. The ready-to-code discretized equation has been coded into NCEP GFS including spectral transform, spectral derivative, semi-Lagrangian and semi-implicit with two-time-level scheme. It is under testing and debugging, we hope to present its preliminary results in the meeting.

2. Deep atmospheric equation set

The deep atmospheric dynamic system has no approximation in Euler equation set, which can be derived into generalized vertical coordinates and written as

$$\frac{du^*}{dt}+\frac{u^*w}{r}-f_s v^*+f_c^* w+\frac{\kappa h}{p}\frac{1}{r}\left(\frac{\partial p}{\partial \lambda}-\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r}\frac{\partial r}{\partial \lambda}\right)=F_u \qquad (2.1)$$

$$\frac{dv^*}{dt}+\frac{v^*w}{r}+f_s u^*+m^2\frac{s^{*2}}{r}\sin\phi+\frac{\kappa h}{p}\frac{1}{r}\left(\frac{\partial p}{\partial \varphi}-\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r}\frac{\partial r}{\partial \varphi}\right)=F_v \qquad (2.2)$$

$$\frac{dw}{dt}-m^2\frac{s^{*2}}{r}-m^2 f_c^* u^*+\frac{\kappa h}{p}\frac{\partial p}{\partial \zeta}\frac{\partial \zeta}{\partial r}+g=F_w \qquad (2.3)$$

$$\frac{\partial \rho^*}{\partial t}+m^2\frac{\partial}{\partial \lambda}\left(\rho^*\frac{u^*}{r}\right)+m^2\frac{\partial}{\partial \varphi}\left(\rho^*\frac{v^*}{r}\right)+\frac{\partial}{\partial \zeta}\left(\rho^*\dot{\zeta}\right)=F_\rho^* \qquad (2.4)$$

$$\frac{dh}{dt}-\frac{\kappa h}{p}\frac{dp}{dt}=F_h \qquad (2.5)$$

$$\frac{dq_i}{dt}=F_{q_i} \qquad (2.6)$$

$$p=\rho\kappa h \qquad (2.7)$$

where

$$\frac{d()}{dt}=\frac{\partial()}{\partial t}+\dot{\lambda}\frac{\partial()}{\partial \lambda}+\dot{\varphi}\frac{\partial()}{\partial \varphi}+\dot{\zeta}\frac{\partial()}{\partial \zeta}=\frac{\partial()}{\partial t}+\frac{m^2u^*}{r}\frac{\partial()}{\partial \lambda}+\frac{m^2v^*}{r}\frac{\partial()}{\partial \varphi}+\dot{\zeta}\frac{\partial()}{\partial \zeta} \qquad (2.8)$$

$$u^*=u\cos\phi=r\cos^2\phi\frac{d\lambda}{dt} \qquad (2.9)$$

$$v^*=v\cos\phi=r\cos\phi\frac{d\phi}{dt} \qquad (2.10)$$

$$w = \frac{dr}{dt} \tag{2.11}$$

$$\rho^* = \rho \frac{r^2}{a^2} \frac{\partial r}{\partial \zeta} \tag{2.12}$$

$$f_s = 2\Omega \sin\phi \tag{2.13}$$

$$f_c^* = 2\Omega \cos^2\phi \tag{2.14}$$

$$s^{*2} = u^{*2} + v^{*2} \tag{2.15}$$

$$g = \bar{g} \frac{a^2}{r^2} \tag{2.16}$$

$$m = \frac{1}{\cos\phi} \tag{2.17}$$

There are three momentum equations with three-dimensional Coriolis force terms, the mass conservation equation with vertical height included. The enthalpy equation as shown in Juang (2011) is used here as well. Note that, momentums, coordinated transformed density, and gravitational force are function of height.

3. Mass coordinate system

In order to consider terrain, the traditional mass coordinates are considered. Instead of using the definition in nonhydrostatic mesoscale spectral model (Juang 2000) to have hydrostatic coordinate (Juang 1992), we start to use mass coordinates. The mass of any given surface by integral of the mass above, we can have

$$M = \int_{\phi_1}^{\phi_2} \int_{\lambda_1}^{\lambda_2} \int_{\zeta}^{\zeta_{TOP}} \rho r^2 \cos\phi \frac{\partial r}{\partial \zeta} d\zeta \, d\lambda \, d\phi \tag{3.1}$$

To utilize the mass convergence equation and this mass definition, we introduce a coordinate pressure as

$$\tilde{\bar{p}} = \frac{M \ \bar{g}}{A_a} = \frac{\int_{\phi_1}^{\phi_2} \int_{\lambda_1}^{\lambda_2} \int_{\zeta}^{\zeta_{TOP}} \rho \bar{g} r^2 \cos\phi \frac{\partial r}{\partial \zeta} d\zeta \, d\lambda \, d\phi}{\int_{\phi_1}^{\phi_2} \int_{\lambda_1}^{\lambda_2} a^2 \cos\phi \frac{\partial r}{\partial \zeta} d\zeta \, d\lambda \, d\phi} = \int_{\zeta}^{\zeta_{TOP}} \rho \bar{g} \frac{r^2}{a^2} \frac{\partial r}{\partial \zeta} d\zeta \tag{3.2}$$

Let coordinate pressure be zero at top of model, then any given surface, coordinate pressure can be written as coordinate pressure gradient integral

$$\tilde{\bar{p}}_\zeta = -\int_{\zeta}^{\zeta_{TOP}} \frac{\partial \tilde{\bar{p}}}{\partial \zeta} d\zeta \tag{3.3}$$

From above two equations and coordinate density of (2.12), we can get coordinate hydrostatic relation in following form as

$$\frac{\partial \tilde{\bar{p}}}{\partial \zeta} = -\rho^* \bar{g} \tag{3.4}$$

Put it into mass conservation equation (2.4), we have

$$\frac{\partial}{\partial t}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta}\right) + m^2 \left(\frac{\partial}{\partial \lambda}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta} \frac{u^*}{r}\right) + \frac{\partial}{\partial \varphi}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta} \frac{v^*}{r}\right) \right) + \frac{\partial}{\partial \zeta}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta} \dot{\zeta}\right) = 0 \tag{3.5}$$

which is similar form as hydrostatic system. Furthermore, the coordinate hydrostatic relation gives us the relation between generalized coordinate and height as

$$\frac{\partial r}{\partial \zeta} = -\frac{a^2}{r^2} \frac{1}{\rho \bar{g}} \frac{\partial \tilde{\bar{p}}}{\partial \zeta} \tag{3.6}$$

Details can be found in Juang (2014).

4. Modeling equation set

We modify the mass-coordinated deep atmospheric equation in the previous section into an alternated form to take advantage of the existed hydrostatic GFS to reduce recoding similar routines, especially the spectral transform routines. First, we let $\varepsilon = \frac{r}{a}$ and wind as following

$$\tilde{u} = a \cos^2\phi \frac{d\lambda}{dt} = \frac{u^*}{\varepsilon} \tag{4.1}$$

$$\tilde{v} = a \cos\phi \frac{d\phi}{dt} = \frac{v^*}{\varepsilon} \tag{4.2}$$

$$\tilde{w} = \varepsilon^2 w \tag{4.3}$$

From total derivatives, we have

$$\frac{d\tilde{u}}{dt} = \frac{d}{dt}\left(\frac{u^*}{\varepsilon}\right) = \frac{1}{\varepsilon}\left(\frac{du^*}{dt} - \frac{u^* w}{\varepsilon a}\right) \tag{4.4}$$

$$\frac{d\tilde{v}}{dt} = \frac{d}{dt}\left(\frac{v^*}{\varepsilon}\right) = \frac{1}{\varepsilon}\left(\frac{dv^*}{dt} - \frac{v^* w}{\varepsilon a}\right) \tag{4.5}$$

$$\frac{d\tilde{w}}{dt} = \frac{d}{dt}\left(w\varepsilon^2\right) = \varepsilon^2 \left(\frac{dw}{dt} + \frac{2w^2}{\varepsilon a}\right) \tag{4.6}$$

In this way, horizontal wind is modified to use shallow definition, and the height-weighted vertical wind can be used to simplify vertical gradient, see Juang (2014), so the modeling equation to use existed spectral transform can be written as

$$\frac{d\tilde{u}}{dt} + 2\frac{\tilde{u}\tilde{w}}{\varepsilon^3 a} - f_s \tilde{v} + f_c^* \frac{\tilde{w}}{\varepsilon^3} + \frac{\kappa h}{p \varepsilon^2} \frac{\partial p}{a \partial \lambda} + \frac{\partial p}{\partial \zeta} \frac{\partial \zeta}{\partial \tilde{\bar{p}}} \frac{\partial \bar{\Phi}}{a \partial \lambda} = F_u \tag{4.7}$$

$$\frac{d\tilde{v}}{dt} + 2\frac{\tilde{v}\tilde{w}}{\varepsilon^3 a} + f_s \tilde{u} + m^2 \frac{\tilde{s}^2}{a} \sin\phi + \frac{\kappa h}{p \varepsilon^2} \frac{\partial p}{a \partial \varphi} + \frac{\partial p}{\partial \zeta} \frac{\partial \zeta}{\partial \tilde{\bar{p}}} \frac{\partial \bar{\Phi}}{a \partial \varphi} = F_v \tag{4.8}$$

$$\frac{d\tilde{w}}{dt} - 2\frac{\tilde{w}^2}{\varepsilon^3 a} - m^2 \varepsilon^3 \frac{\tilde{s}^2}{a} - m^2 \varepsilon^3 f_c^* \tilde{u} - \bar{g}\left(\varepsilon^4 \frac{\partial p}{\partial \zeta} \frac{\partial \zeta}{\partial \tilde{\bar{p}}} - 1\right) = F_w \tag{4.9}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta}\right) + m^2 \left(\frac{\partial}{a \partial \lambda}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta} \tilde{u}\right) + \frac{\partial}{a \partial \varphi}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta} \tilde{v}\right) \right) + \frac{\partial}{\partial \zeta}\left(\frac{\partial \tilde{\bar{p}}}{\partial \zeta} \dot{\zeta}\right) = 0 \tag{4.10}$$

And other equations are no changed, which are (2.5), (2.6) and (2.7), and where

$$\frac{d()}{dt} = \frac{\partial()}{\partial t} + \dot{\lambda} \frac{\partial()}{\partial \lambda} + \dot{\varphi} \frac{\partial()}{\partial \varphi} + \dot{\zeta} \frac{\partial()}{\partial \zeta} = \frac{\partial()}{\partial t} + m^2 \tilde{u} \frac{\partial()}{a \partial \lambda} + m^2 \tilde{v} \frac{\partial()}{a \partial \varphi} + \dot{\zeta} \frac{\partial()}{\partial \zeta} \tag{4.11}$$

$$\bar{\Phi} = \bar{g} r \tag{4.12}$$

We can find there are shallow atmospheric derivatives for all derivatives with modification of u to be shallow atmospheric definition and deep atmospheric parameter ε, otherwise there is no r, and horizontal winds are in shallow atmospheric definition. We can use existed spectral transform routines, especially between horizontal wind, divergence, and vorticity as usual. Nevertheless, the entire equation is still a deep atmospheric dynamic system.

5. Initial condition preparation

Since it may take a long time to develop future data assimilation system by using deep atmospheric dynamics, we will use current hydrostatic dynamics as initial condition to the deep-atmospheric system. However, several steps are required for further considerations. First, we use the same definition of coordinates constants to define vertical coordinate by coordinate pressure, because coordinate pressure is hydrostatic relation to height as monotone, thus we have

$$\tilde{\tilde{p}} = A + B\tilde{\tilde{p}}_s \qquad (5.1)$$

where A and B as the same as hydrostatic system used in Juang (2011). We use the definition of coordinate pressure to compute the location of hydrostatic initial condition, then interpolate into the new definition by A and B for deep-atmospheric system as initial condition.

Another concern is related to hydrostatic relation in deep-atmospheric system. Since g is function of r, the hydrostatic balance in deep-atmospheric dynamic should be written as

$$\frac{\partial p}{\partial r} = -\rho g = -\rho \frac{a^2}{r^2} \bar{g} \qquad (5.2)$$

where g is not constant but function of r as shown. Thus, from the vertical momentum equation and hydrostatic balance, we should have following relation

$$\frac{\Delta \tilde{\tilde{p}}}{\Delta p} = \varepsilon^4 \qquad (5.3)$$

So, when we define coordinate pressure and the location in term of r, we can get hydrostatic balanced pressure by the relation above (5.3) and coordinate pressure definition in section 3 equation (3.6) by iteration for entire atmosphere in vertical direction at any given location for initial condition as hydrostatic balance for deep atmospheric dynamics.

6. Conclusion

Implement deep-atmospheric system into operational forecasting system should not have any practical problem, because UK Meteo Office had done it even about a decade ago (Davies et al 2005). In NCEP, we are somewhat slower than other centers on advancing development of global model dynamics. Instead of moving a reasonable step from hydrostatic system to nonhydrostatic system as most of centers are doing, we follow UK Meteo Office to advance further step to request an implementation to reach non-approximation Euler equation set for global forecasting system, not only to avoid future dynamical change but also to support SWPC in NCEP.

This extended abstract has illustrated what we will use on implementation of deep-atmospheric Euler equation set for global forecasting system, which including full dimensional Coriolis force and full dimension of momentum equation as nonhydrostatic system. We hope it could support dynamic system for a long period of time because it covers all scales and whole atmosphere for coupling other components of Earth modeling system.

References

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