

中央氣象局全球預報系統的現況與未來

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1. **CWBGFS現況**
2. **CWBGFS未來的發展**
3. **結論**

國際各作業中心模式現況

名稱	解析度	水平	垂直座標 層數	模式架構
ECMWF (2010/1)	TL1279L91	16km	S-P 91	Semi_L+spectrum
JMA (2007/11)	TL959L60	20km	S-P 60	Semi_L+spectrum
NCEP(2010/7)	T574L64	23km	S-P 64	spectrum
UKM (2010/5)	N512L70	25km	S-P 70	Semi_L+Finite difference
KMA (韓國2010/5)	N512L70	25km	S-P 70	Semi_L+Finite difference
CMA (中國2008/1)	TL639L60	32km	S-P 60	Semi_L+spectrum
GEM (加拿大2006/8)	(800X600)	34km	S-P 80	Semi_L+finite element
NOGAPS (2009/9)	T319L42	42km	S-P 42	spectrum
CWB (2011/11)	T319L40	42km	S 42	spectrum

CWB全球預報系統- 現況

1. 分析模組: **GSI**
2. 模式動力架構:
 - a. **T319L40** (42km, **sigma**坐標, model top=**1mb**) (2011)
 - b. **T319L60** (42km, **sigma-pressure hybrid**坐標, model top=**0.1mb**) (2012)
3. **Bogus typhoon** (typhoon **relocation**)
4. 模式物理參數化:
 - a. **土壤模式**: 參考Mahrt and Pan (1984) 之二層土壤模式。
 - b. **地表通量參數化**: 參考Businger et al. (1971)。
 - c. **大氣垂直紊流混合參數化**: 考慮非局部紊流通量之一階閉合邊界層參數法(Troen and Mahrt 1986)。
 - d. **重力波拖曳參數化**: 依據Palmer et al. (1986)。
 - e. **積雲對流參數化**: Pan and Wu (1995) 的簡化A-S積雲對流參數法。
 - f. **淺積雲對流參數化**: 採用Li (1994)、Li and Wang (2000)。
 - g. **網格尺度降水參數化**: 採用Zhao and Frederick(1997) 預報雲水(雲冰), 並透過雲物理過程決定降水。
 - h. **輻射參數化**: 短波及長波輻射模式均建立在二向式(two-stream)之計算結構, 但長波方面使用Fu et al. (1997) 的二向\四向混合法以提高其精確性。關於大氣氣體吸收係數與雲光學特性的參數化乃參考Fu and Liou (1992; 1993)。
 - i. **Raylei-friction**: 平流層-U, V

GSI 同化的觀測資料

觀測類別		內容
傳統觀測	地面觀測	SYNOP, SHIP, METAR, BUOY
	高空觀測	TEMP, PILOT, AIREP, PROFILER, NEXTRA
衛星觀測	衛星反演風	SATOB (GOES, GMS, METEOSAT, MTSAT), MODIS (AQUA, TERRA)
	Scaterometer	ASCAT, WINDSAT
	GPSRO	GPSRO (COSMIC, METOP-A/GRAS, GRACE-A, TERRASAR-X)
	SOUNDERS	AMSUA (NOAA15/18/19, METOP-A, AQUA), AMSUB/MHS (NOAA15/18/19/METOP-A), HIRS3/4 (NOAA17/19, METOP), AIRS (AQUA), IASI (METOP-A), AMSRE (AQUA), SNDR (GOES11/12)
	IMAGERS	IMAGERS (GOES11/12)
	OZONE	OBSUV8 (NOAA17/18), OMI (AURA)
	precipitable water	SSMI, TMI

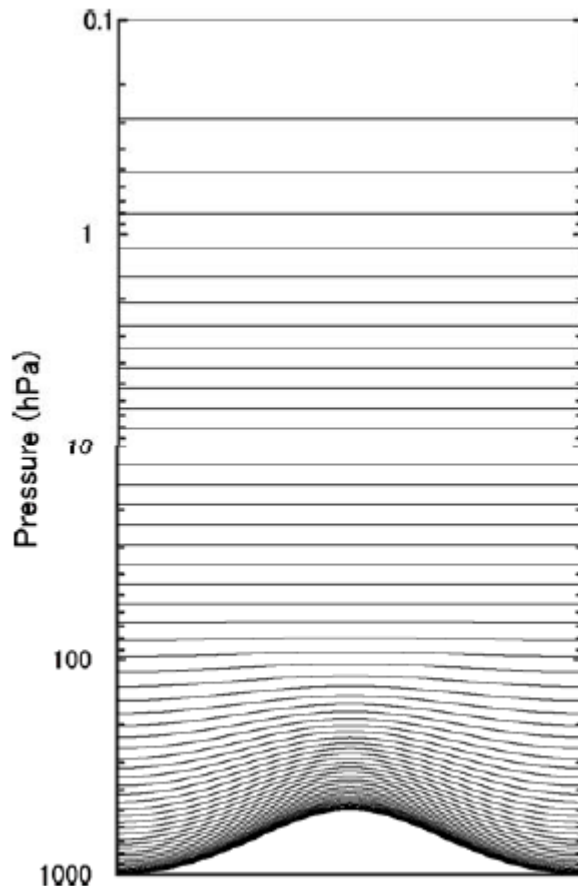
— operational used

— not used

2. 動力架構:

b. T319L60 : Sigma-P hybrid 坐標

Choice of Vertical levels (model top=0.1Mb 約65Km)

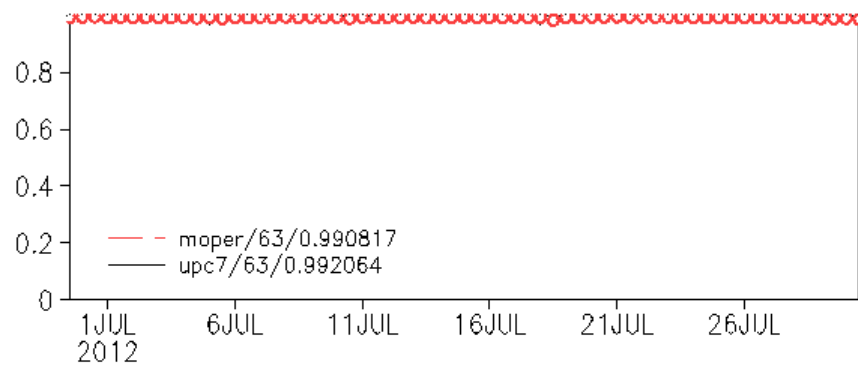


ECMWF-TL512L60: 100 Mb以上有25層

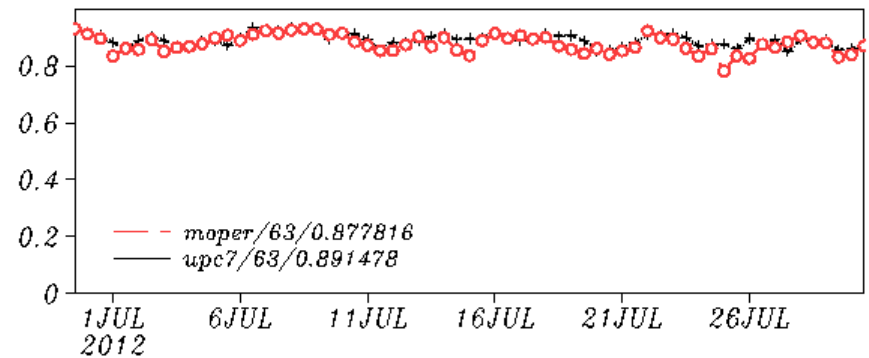
JMA-TL959L60: 100 Mb以上有25層

CWB-T319L60 : 100 Mb以上有23層

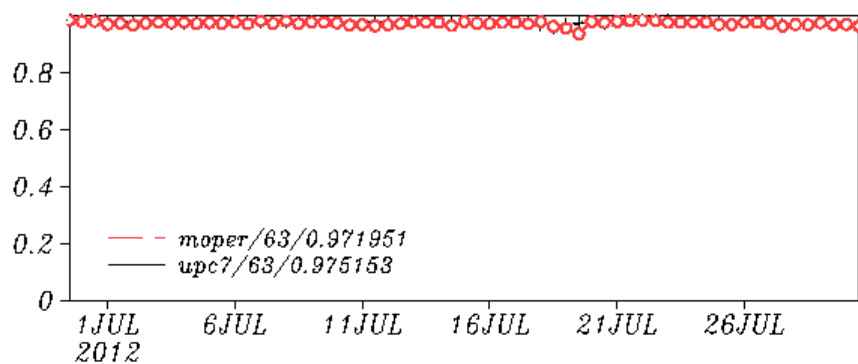
500hPa ach 1 day fcst - NA
2012:6:29:12-2012:7:30:12, 0012Z



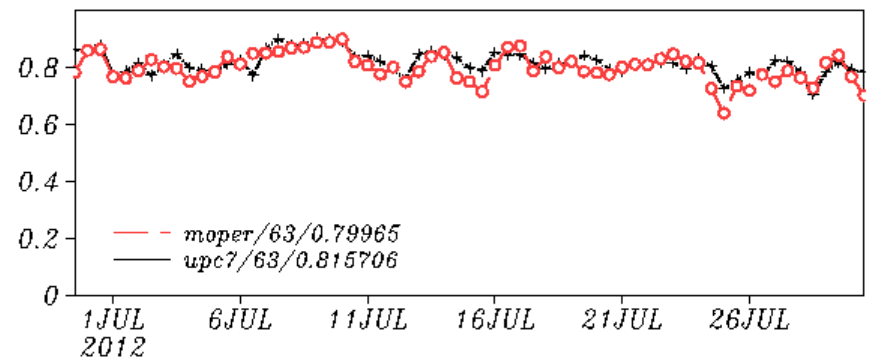
500hPa ach 4 day fcst - NA
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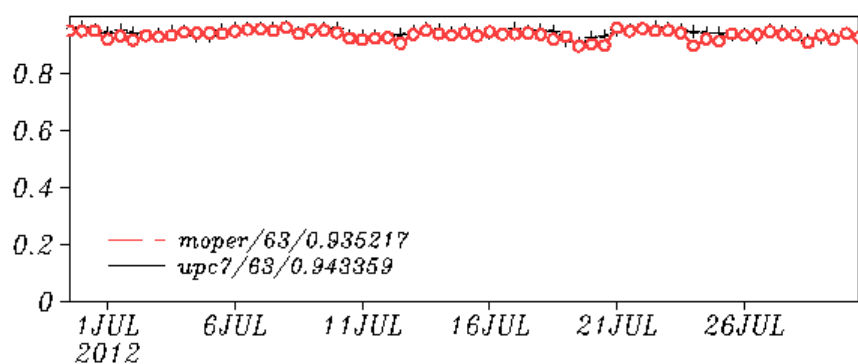
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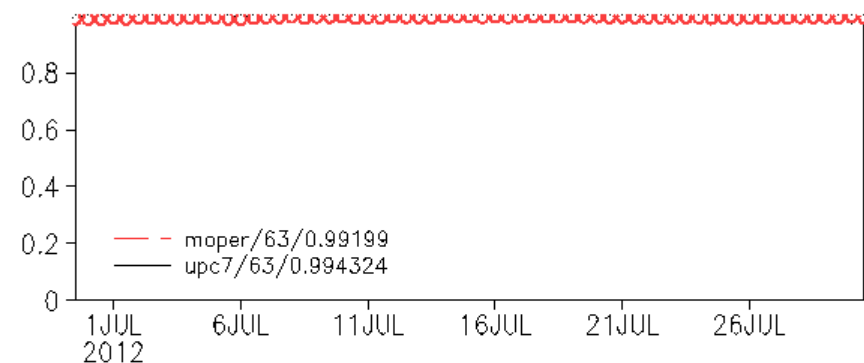
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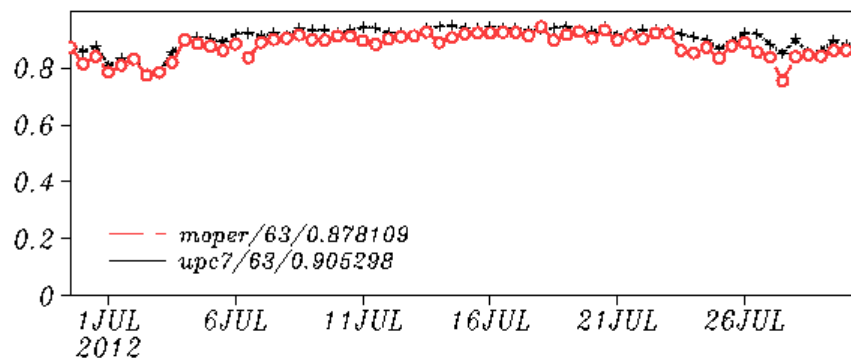
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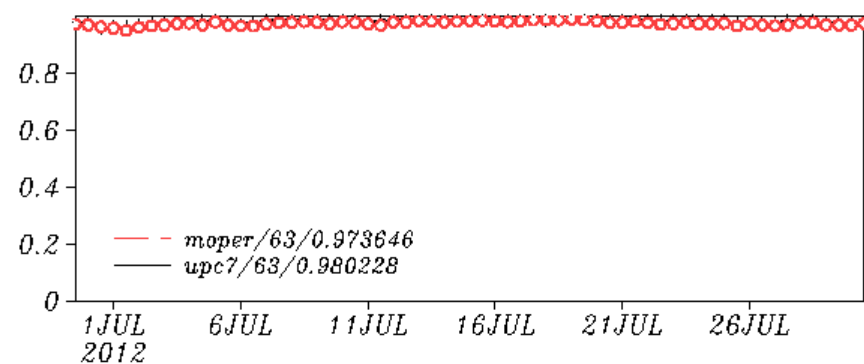
500hPa ach 1 day fest - SA
2012:6:29:12-2012:7:30:12, 0012Z



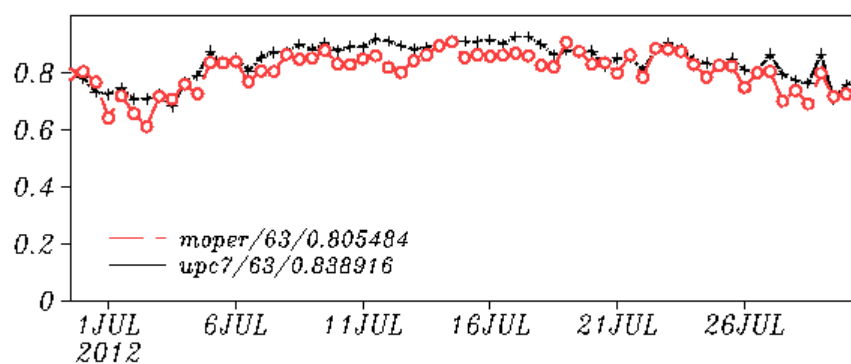
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2012:6:29:12-2012:7:30:12, 0012Z



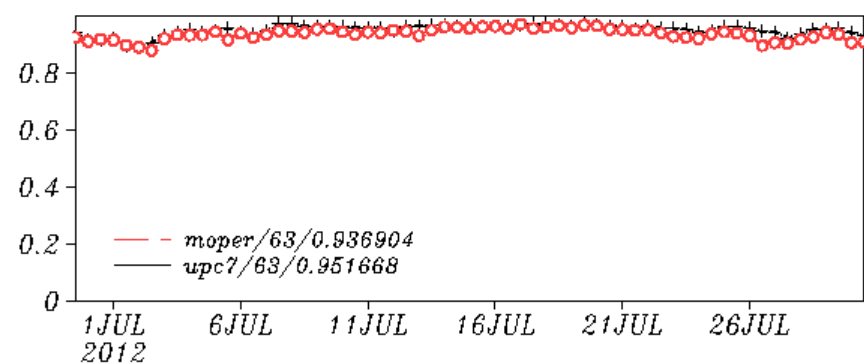
500hPa ach 2 day fest - SA
2012:6:29:12-2012:7:30:12, 0012Z



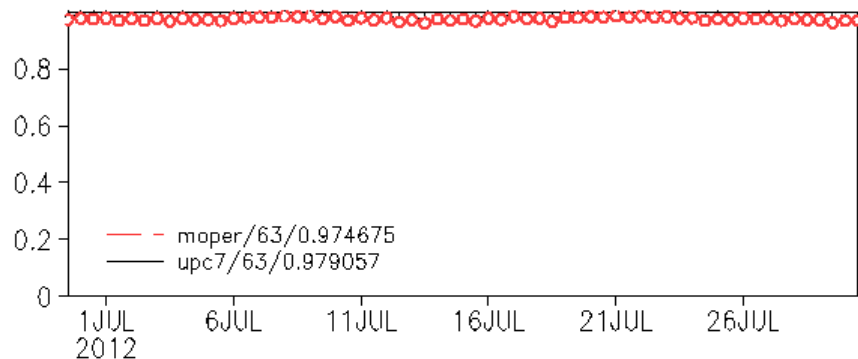
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2012:6:29:12-2012:7:30:12, 0012Z



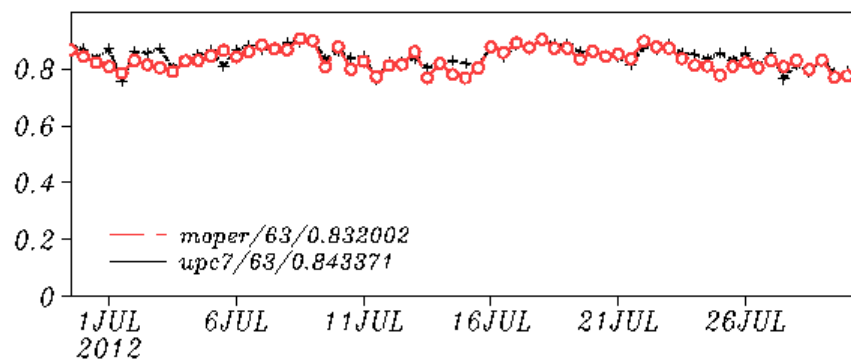
500hPa ach 3 day fest - SA
2012:6:29:12-2012:7:30:12, 0012Z



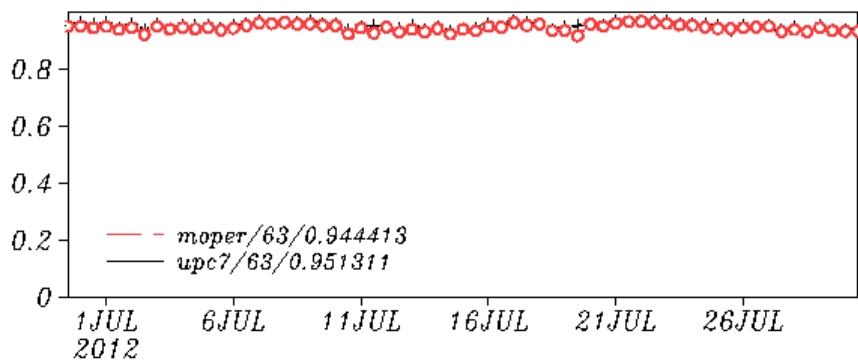
SLP ach 1 day fcast - NA
2012:6:29:12-2012:7:30:12, 0012Z



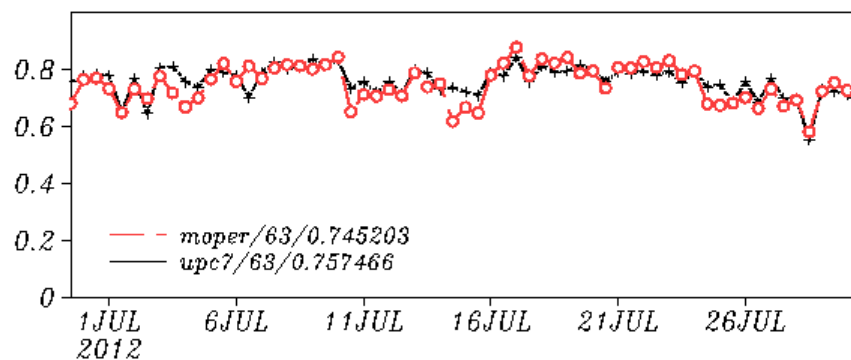
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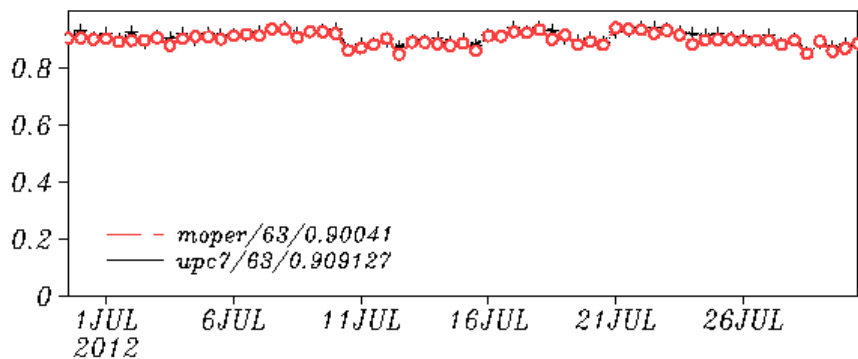
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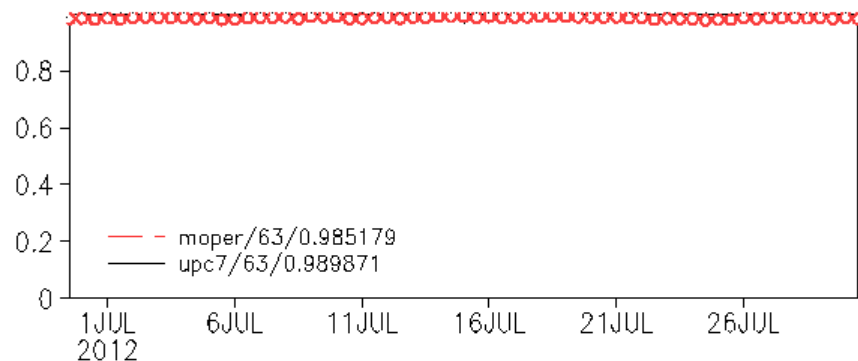
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2012:6:29:12-2012:7:30:12, 0012Z



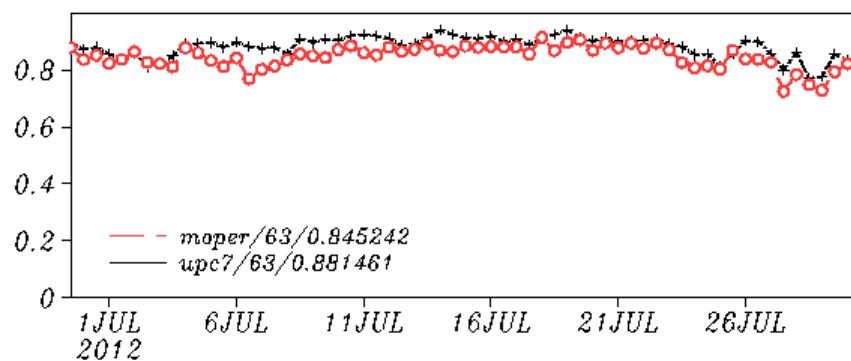
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2012:6:29:12-2012:7:30:12, 0012Z



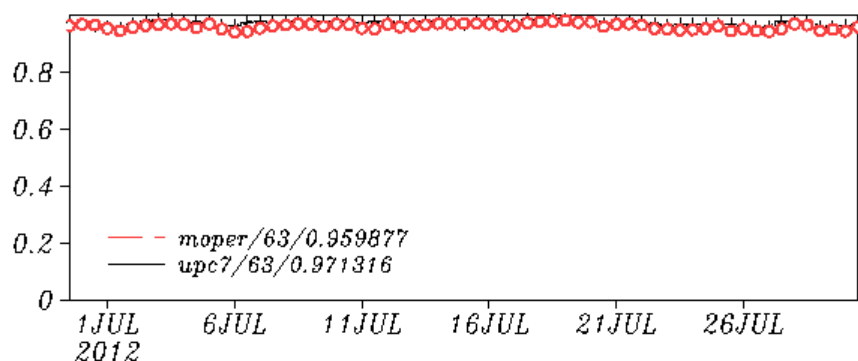
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2012:6:29:12-2012:7:30:12, 0012Z



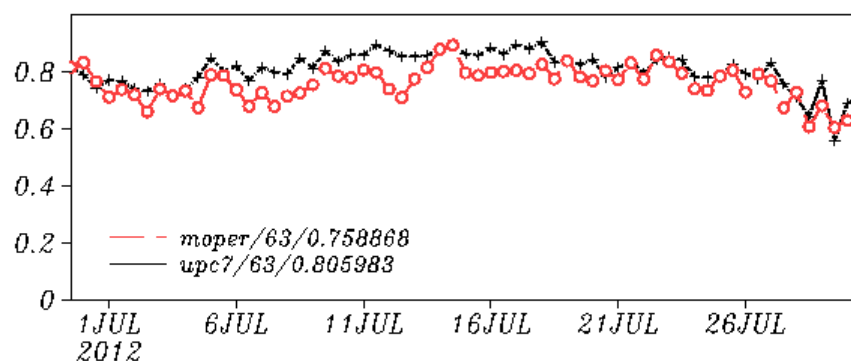
SLP ach 4 day fcst - SA
2012:6:29:12-2012:7:30:12, 0012Z



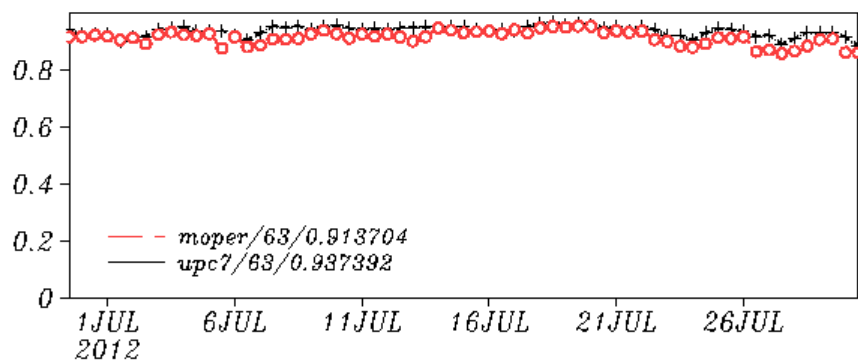
SLP ach 2 day fcst - SA
2012:6:29:12-2012:7:30:12, 0012Z



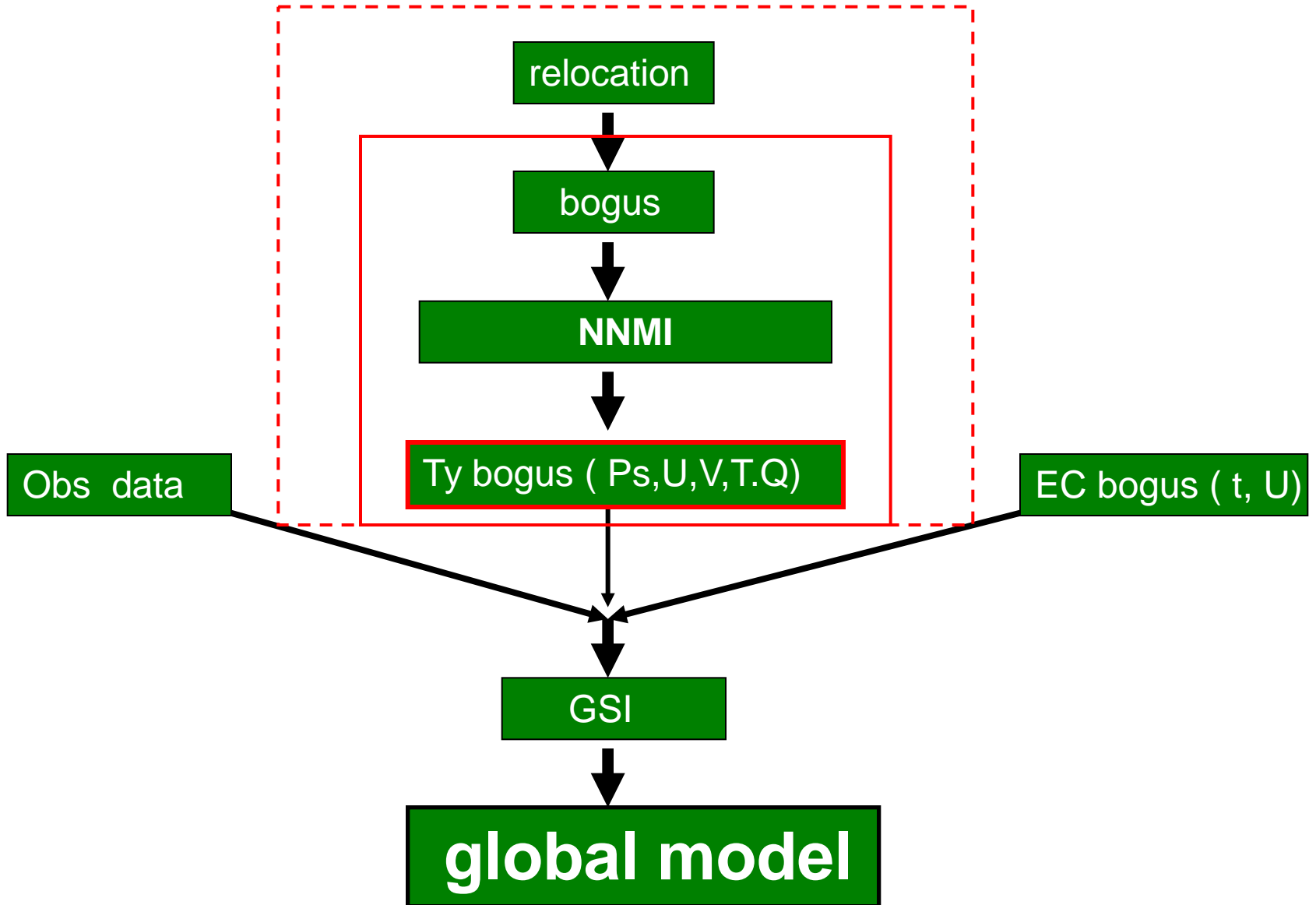
SLP ach 5 day fcst - SA
2012:6:29:12-2012:7:30:12, 0012Z



SLP ach 3 day fcst - SA
2012:6:29:12-2012:7:30:12, 0012Z



CWB GFS



海上陸上颱風警報

中央氣象局 民國 92 年編號第 11 號颱風警報 第 1 報

8 月 19 日 12 時 30 分發布

颱風強度及命名：輕度颱風，國際命名：VAMCO，中文譯名：梵高。

中心氣壓：998 百帕。

中心位置：19 日 11 時的中心位置在北緯 21.6 度，東經 125.2 度，即在花蓮的東南方約 460 公里之海面上。

暴風半徑：七級風暴風半徑 100 公里，十級風暴風半徑 一 公里。

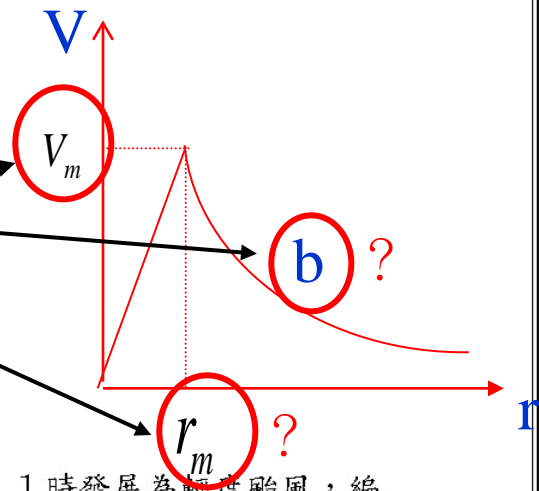
預測速度及方向：以每小時 26 公里速度，向西北進行。

近中心最大風速：每秒 18 公尺(約每小時 65 公里)，相當於 8 級風。

瞬間之最大陣風：每秒 25 公尺(約每小時 90 公里)，相當於 10 級風。

預測位置：20 日 11 時的中心位置在北緯 25.5 度，東經 121.0 度，即在台北的西北方約 80 公里之海面上。

颱風動態：原位於台灣東南方海面之熱帶性低氣壓，已於今(19)日 11 時發展為輕度颱風，編



$$V(r) = V_m \left(\frac{r}{r_m}\right) \exp\left\{\frac{1}{b} \left[1 - \left(\frac{r}{r_m}\right)^b\right]\right\}$$

此移動，對花蓮以

北及新竹以北地區將構成威脅。

V_m ：近中心最大風速
 b ：外圍風速曲率
 r_m ：近中心最大風速半徑

警戒區域及事項：陸上

及強風。

海上

士海峽、台灣海峽北部及

馬祖海面航行及作業船隻應嚴加戒備。

下次警報預定發布時間：8 月 19 日 14 時 30 分。

◎ bogus Algorithm

1. 尋找初始場颱風並決定
移除與植入的颱風的範圍

2. 移除初始場颱風
--- *vor* · *div*

3. 植入颱風

依強度及大小設定初始 r_m, b

$$\bar{V}(r) = \bar{V}_m \left(\frac{r}{r_m} \right) \exp \left\{ \frac{1}{b} \left[1 - \left(\frac{r}{r_m} \right)^b \right] \right\}$$

$$\nabla^2 \left[\phi + \frac{1}{2} (\nabla \psi)^2 \right] = \nabla \cdot [(f + \nabla^2 \psi) \nabla \psi]$$

$$\ln P_s \approx \ln P_* - \frac{\phi}{RT_a}$$

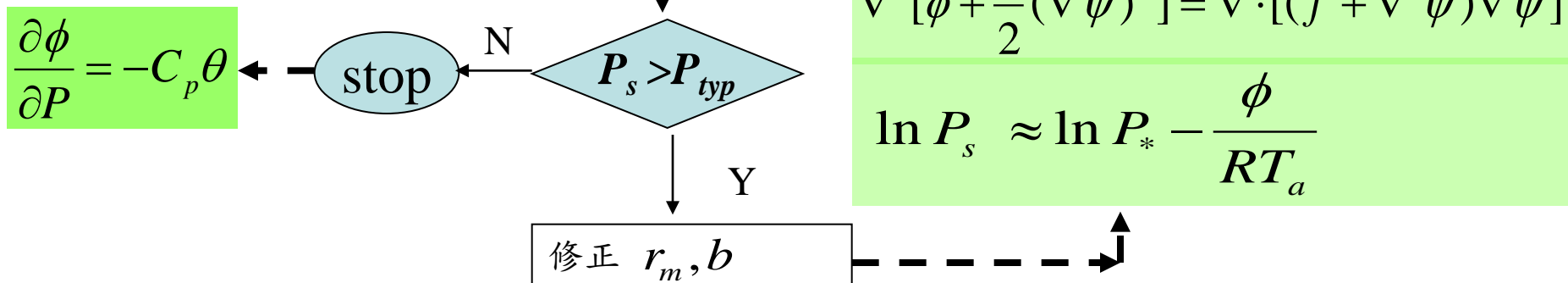
$$\frac{\partial \phi}{\partial P} = -C_p \theta$$

stop

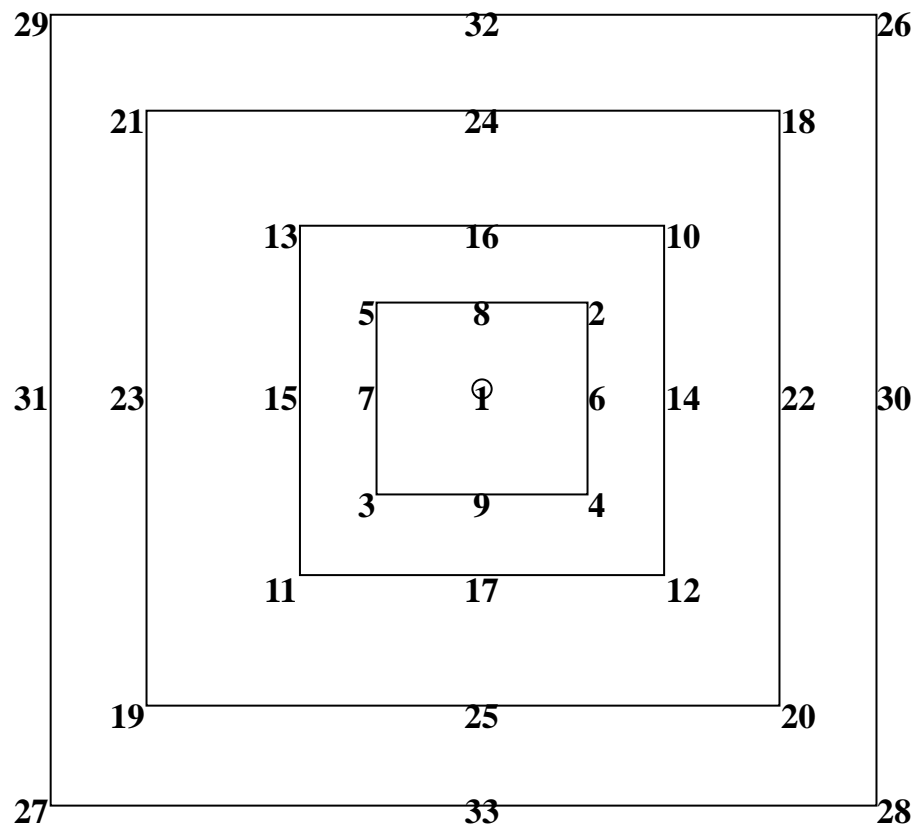
$P_s > P_{typ}$

Y

修正 r_m, b



以颱風中心向外，取6圈（每隔1、2、4、6、8、10網格點取1圈，每圈8個點）共49（含中心點）點



T320L40(S) vs T320L60(S-P)

2012颱風路徑模擬結果

(10Cases)

(13Cases)

SANVU (052200~052718)

MAWAR (060106~060600)

GUCHOL (061206~061918)

TALIM (061718~062018)

DOKSURI (062612~062918)

KHANUN (071606~071818)

VICENTE (072112~072412)

SAOLA (072800~080306)

DAMREY (072818~080300)

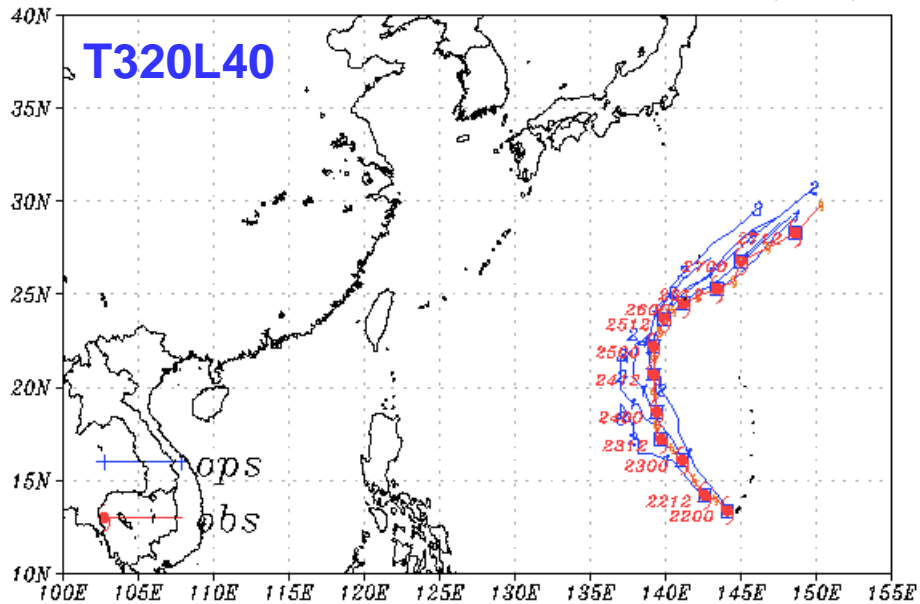
HAIKUI (080300~080906)

KAI-TAK(081300~081800)

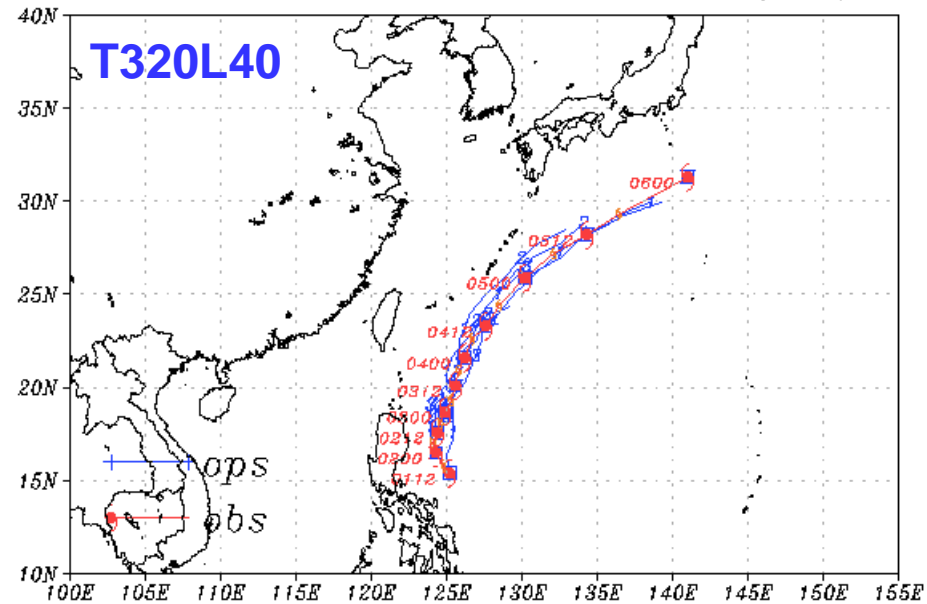
BOLAVEN(082006~082812)

TEMBIN(081900~083006)

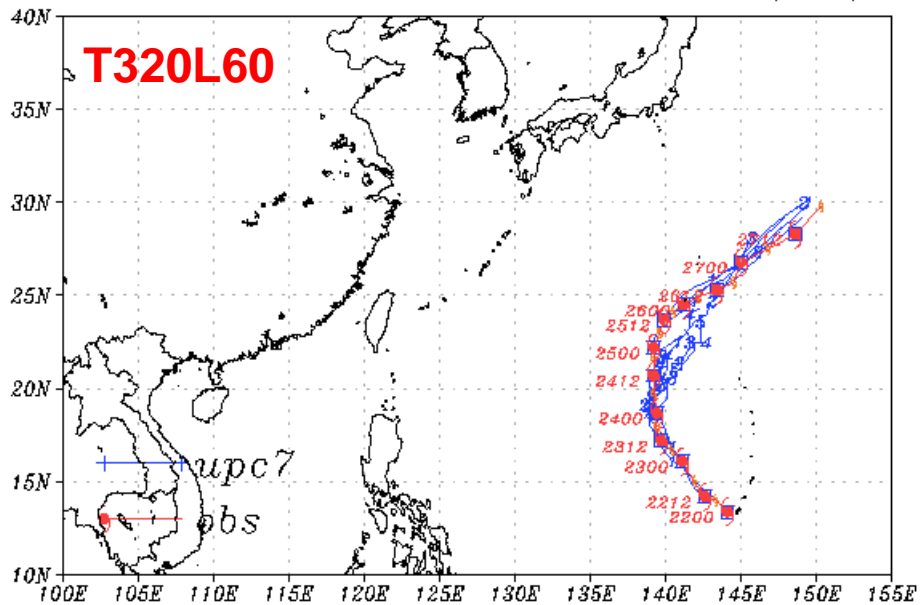
SANVU 2012:5:22:0-2012:5:27:12 (SLP)



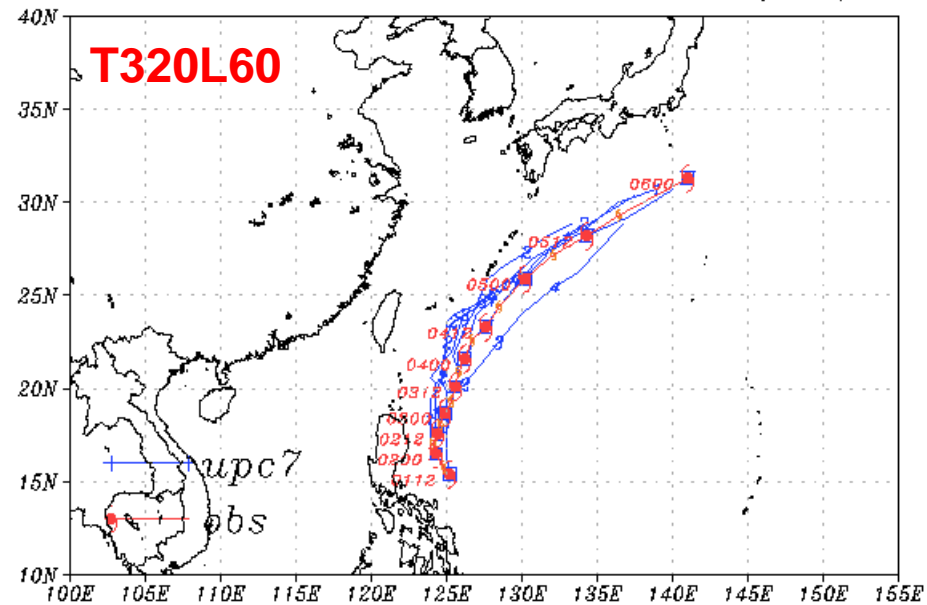
MAWAR 2012:6:1:12-2012:6:6:0 (SLP)



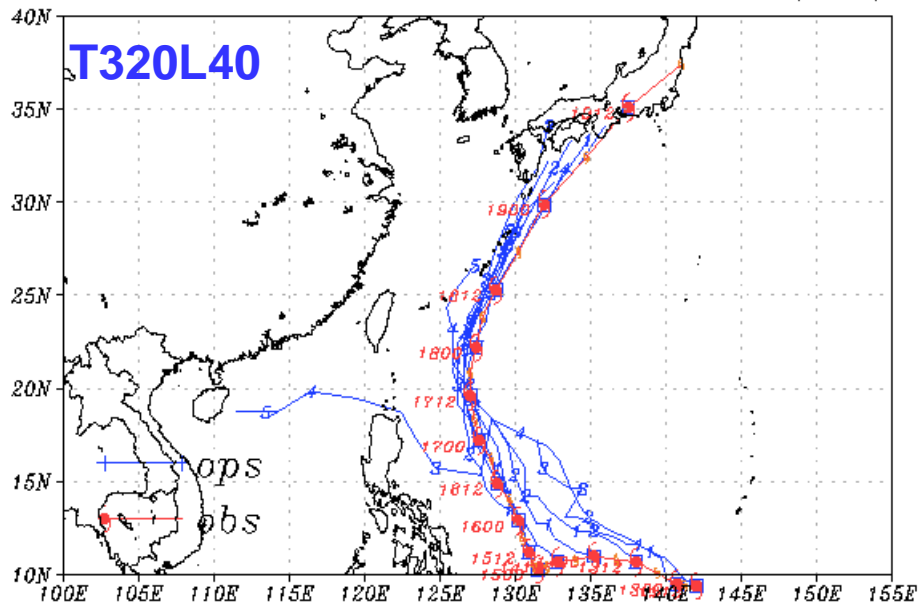
SANVU 2012:5:22:0-2012:5:27:12 (SLP)



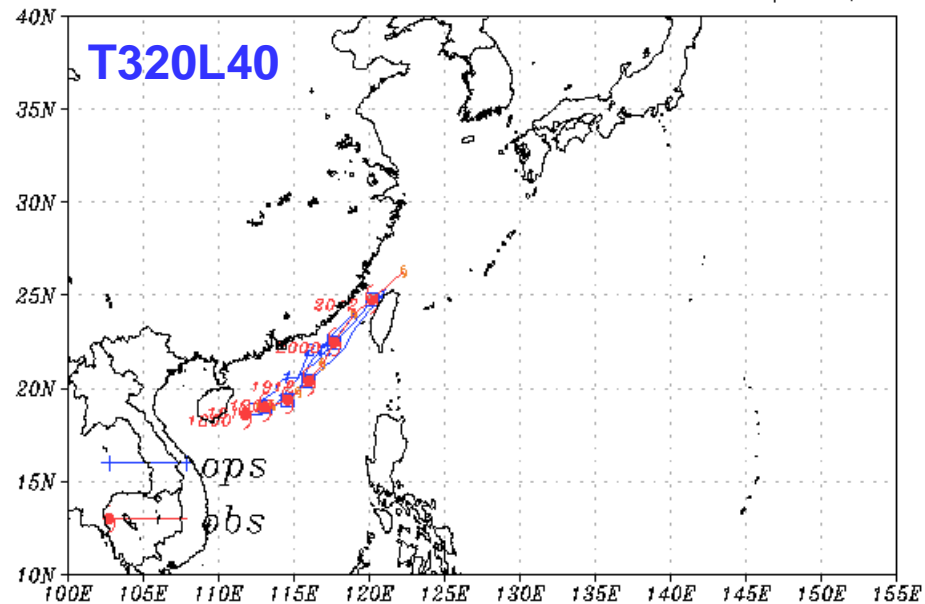
MAWAR 2012:6:1:12-2012:6:6:0 (SLP)



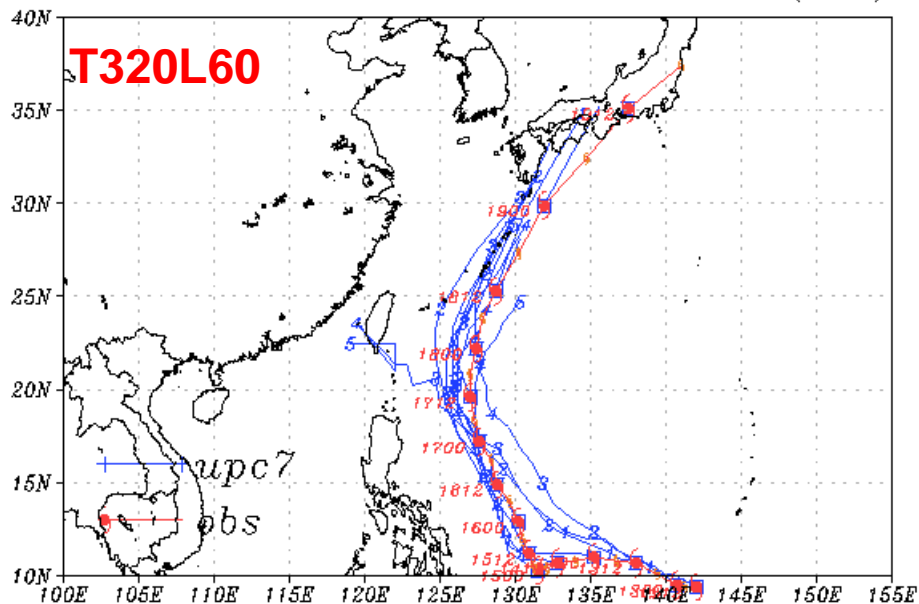
GUCHOL 2012:6:12:12-2012:6:19:12 (SLP)



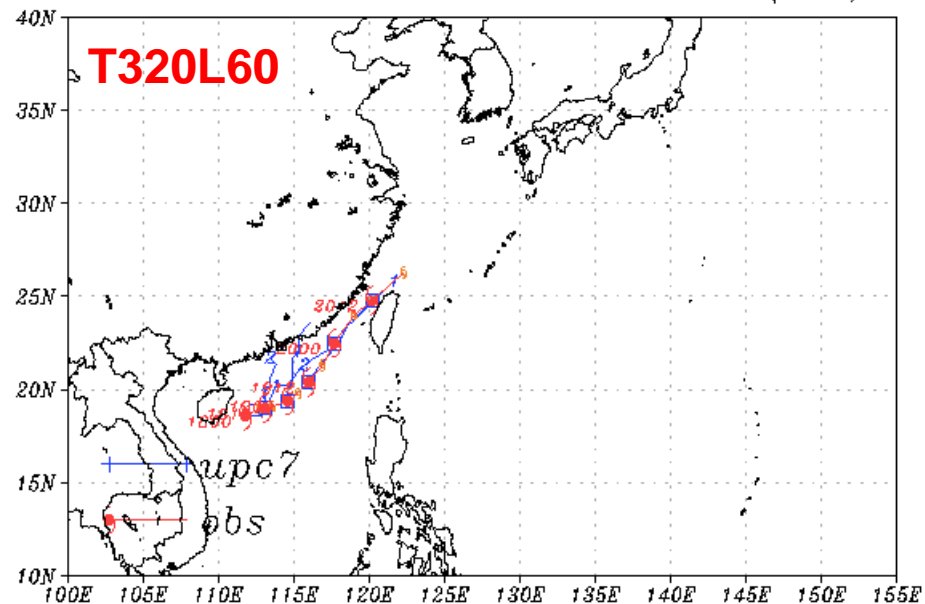
TALIM 2012:6:18:0-2012:6:20:12 (SLP)



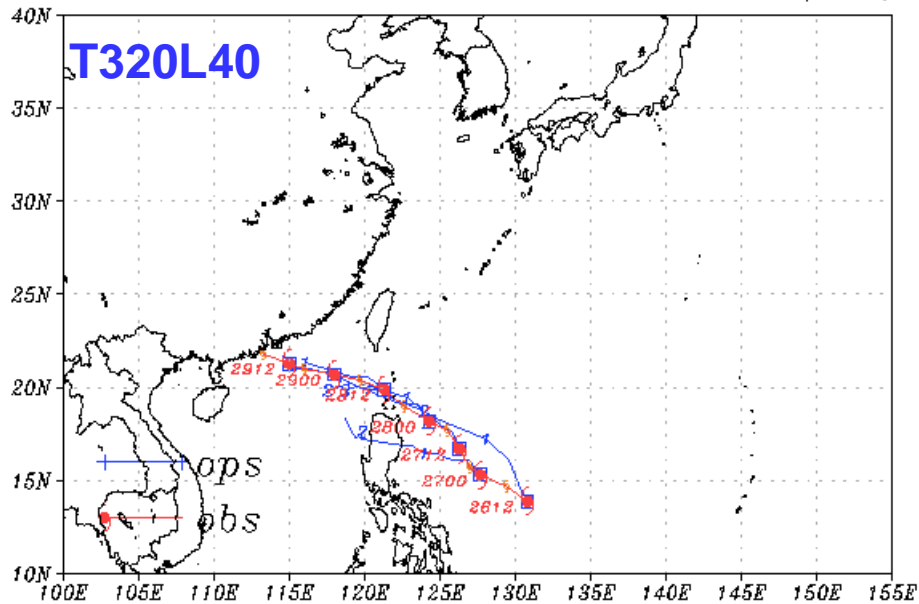
GUCHOL 2012:6:12:12-2012:6:19:12 (SLP)



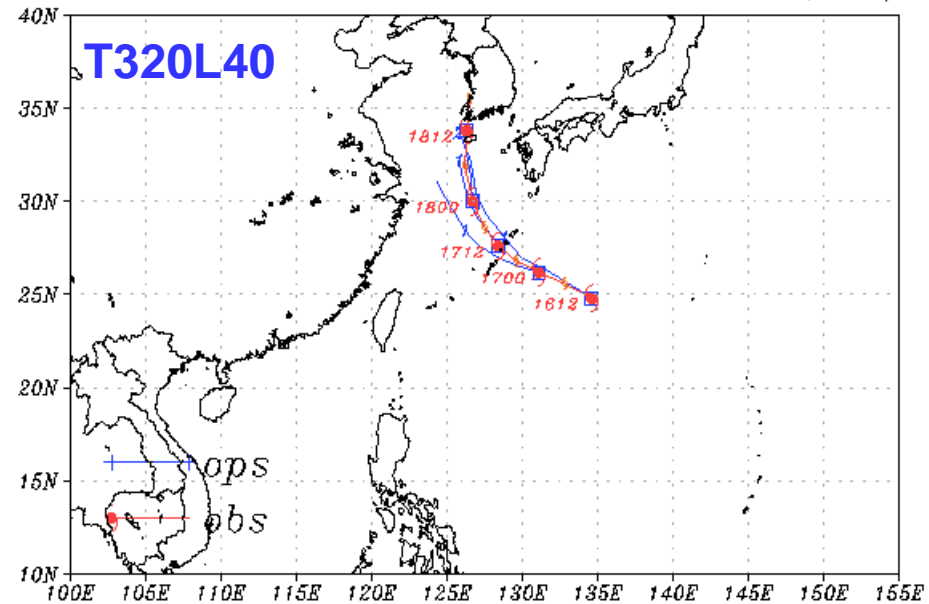
TALIM 2012:6:18:0-2012:6:20:12 (SLP)



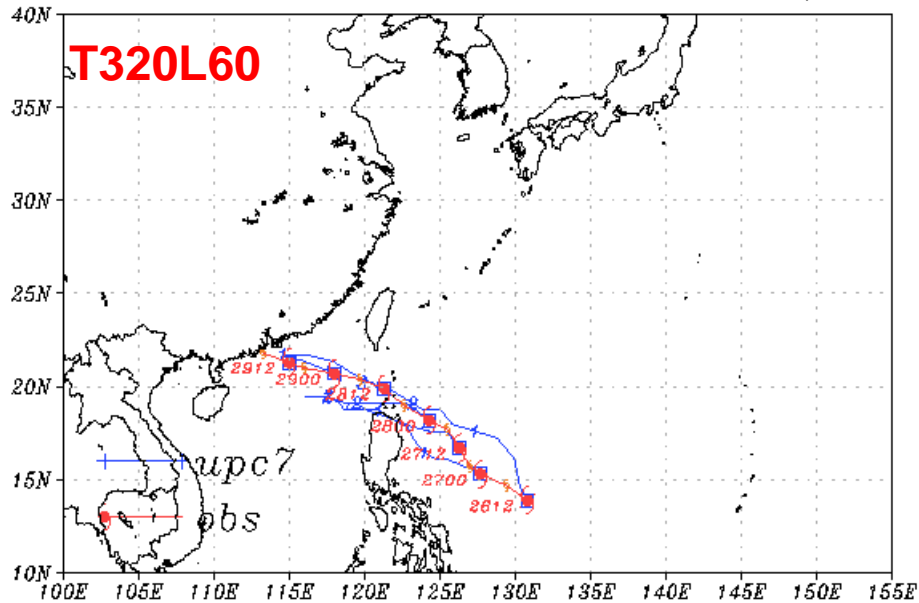
DOKSURI 2012:6:26:12-2012:6:29:12 (SLP)



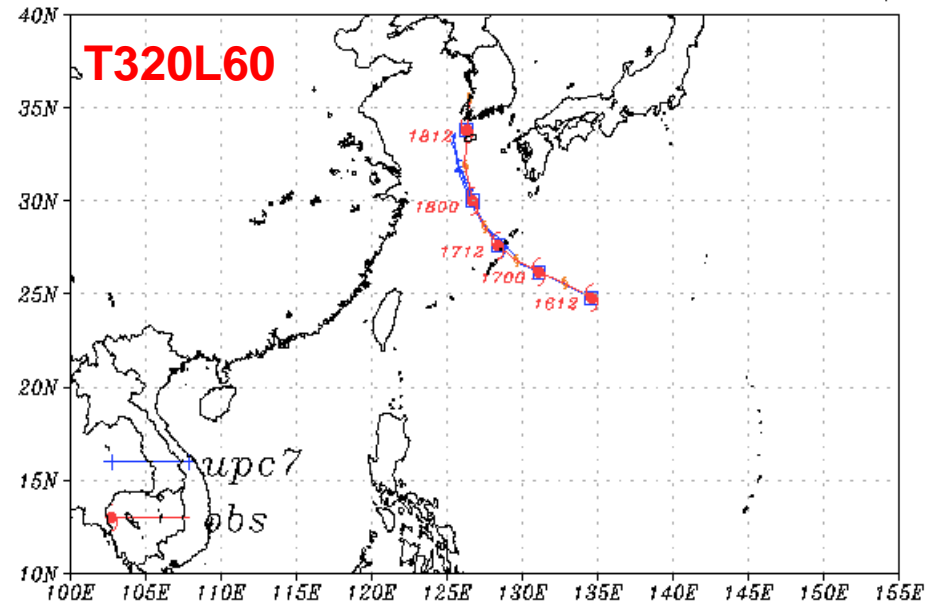
KHANUN 2012:7:16:12-2012:7:18:12 (SLP)



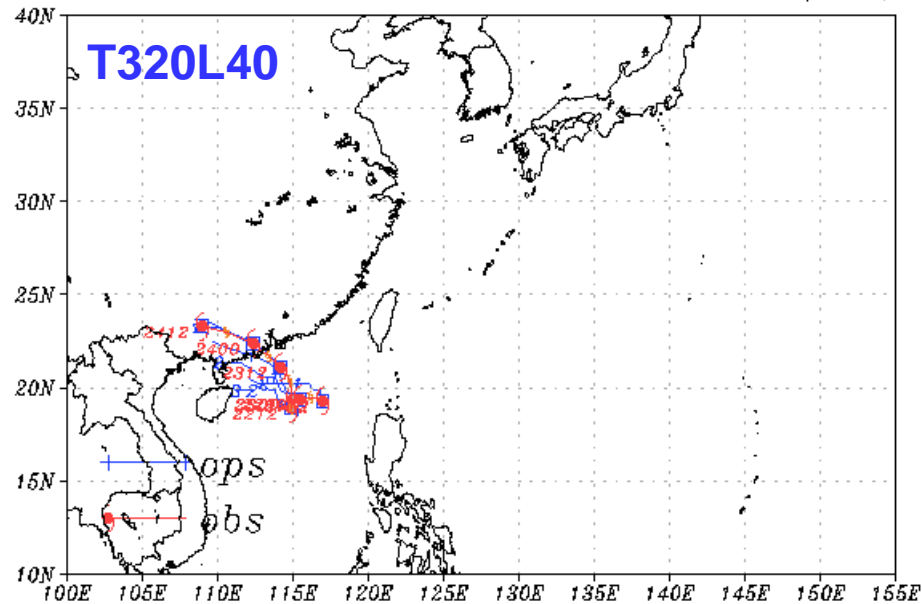
DOKSURI 2012:6:26:12-2012:6:29:12 (SLP)



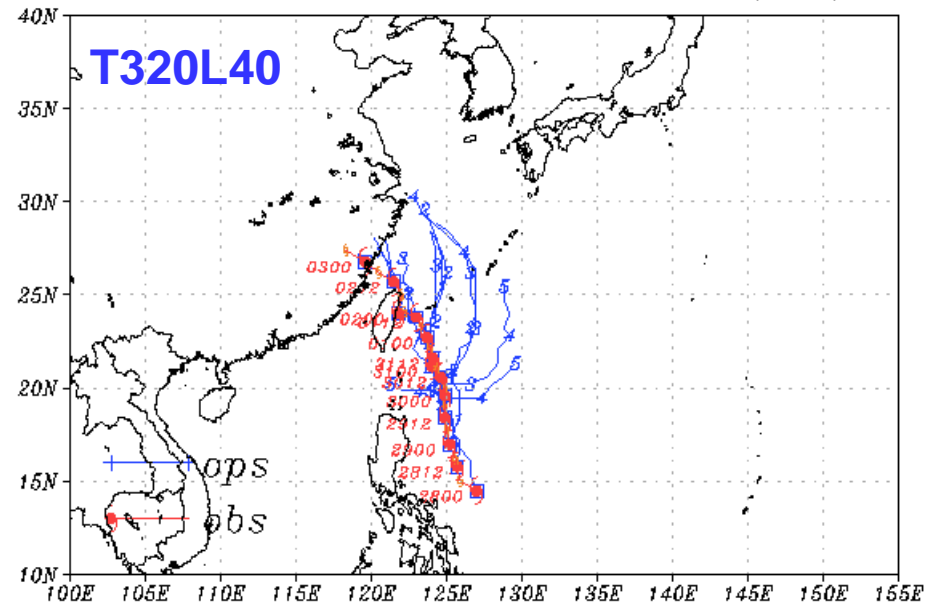
KHANUN 2012:7:16:12-2012:7:18:12 (SLP)



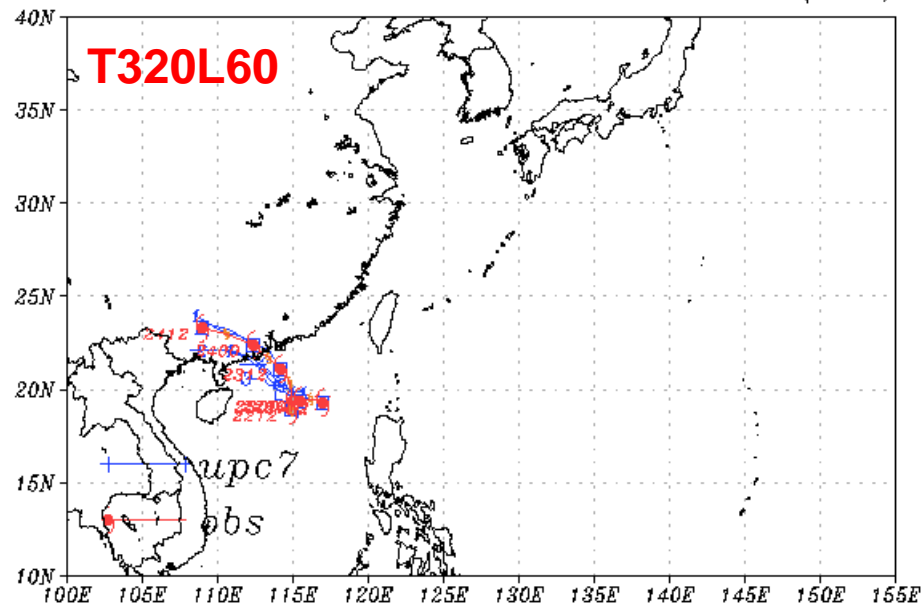
VICENTE 2012:7:21:12-2012:7:24:12 (SLP)



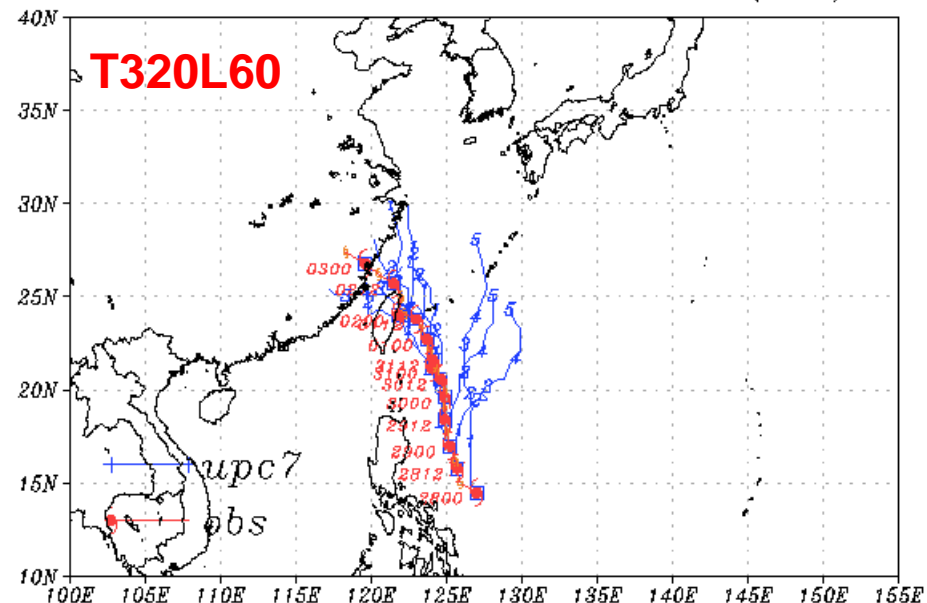
SAOLA 2012:7:28:0-2012:8:3:0 (SLP)



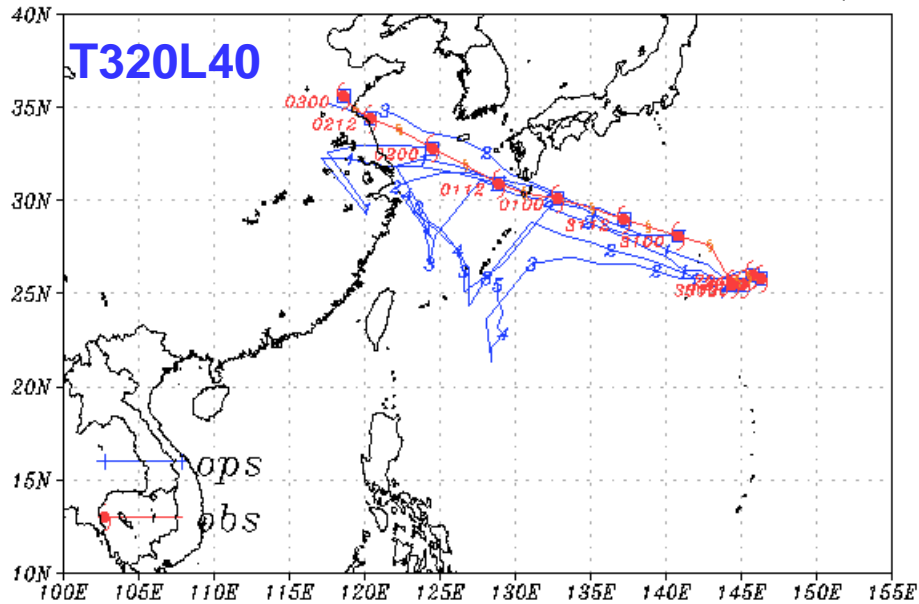
VICENTE 2012:7:21:12-2012:7:24:12 (SLP)



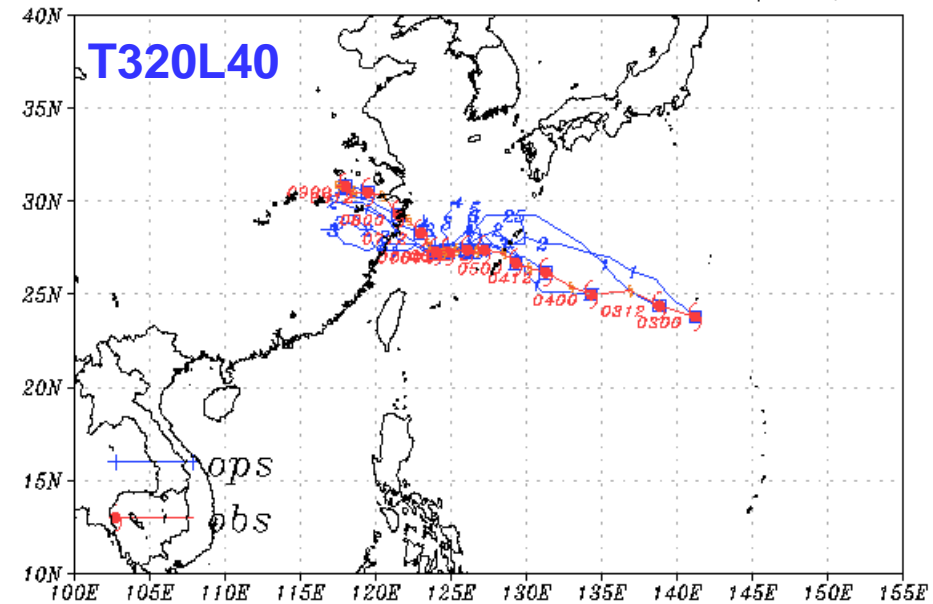
SAOLA 2012:7:28:0-2012:8:3:0 (SLP)



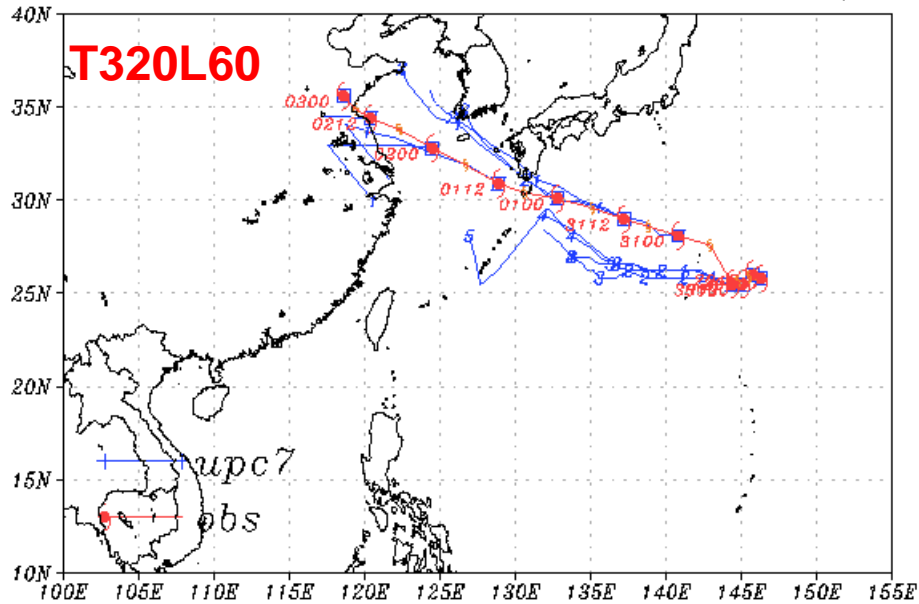
DAMREY 2012:7:29:0-2012:8:3:0 (SLP)



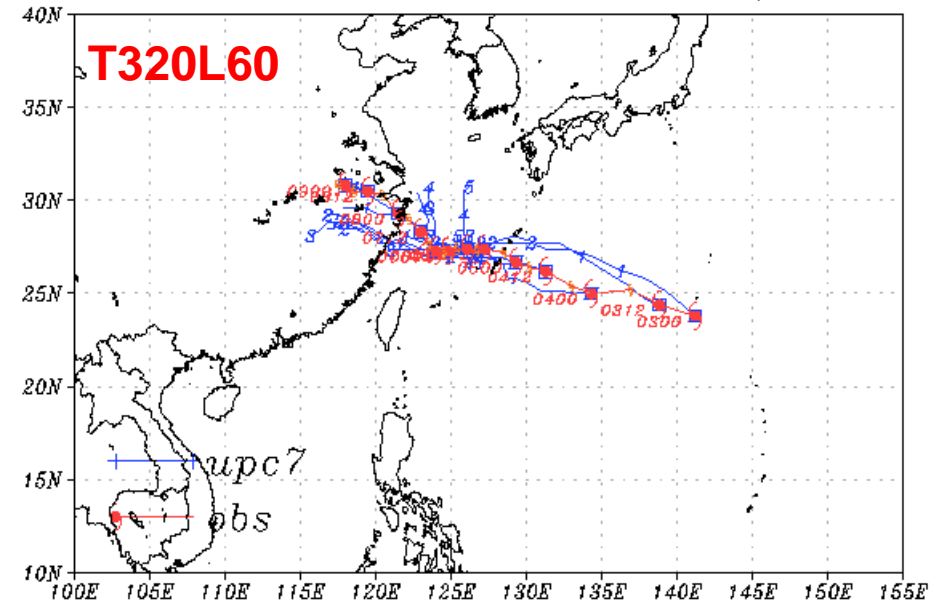
HAIKUI 2012:8:3:0-2012:8:9:0 (SLP)



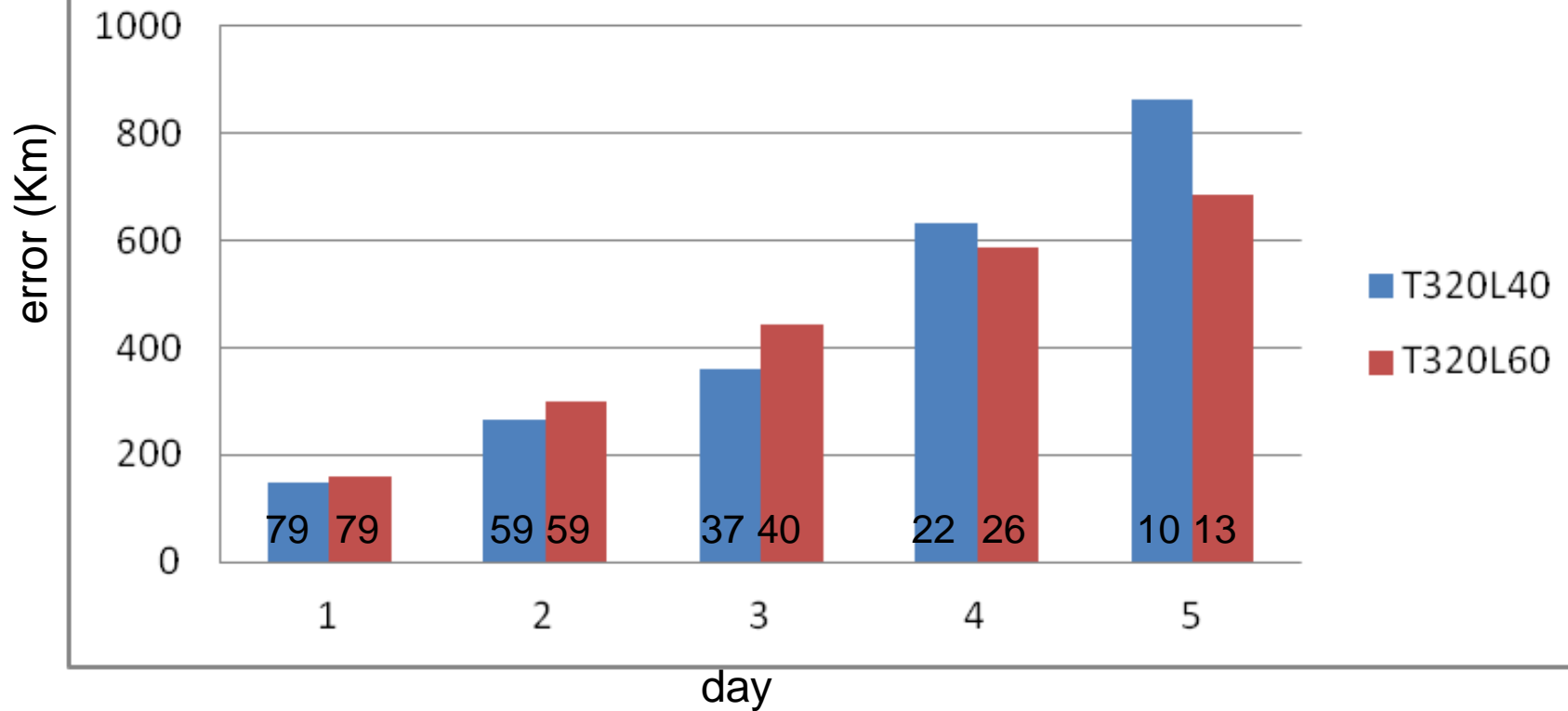
DAMREY 2012:7:29:0-2012:8:3:0 (SLP)



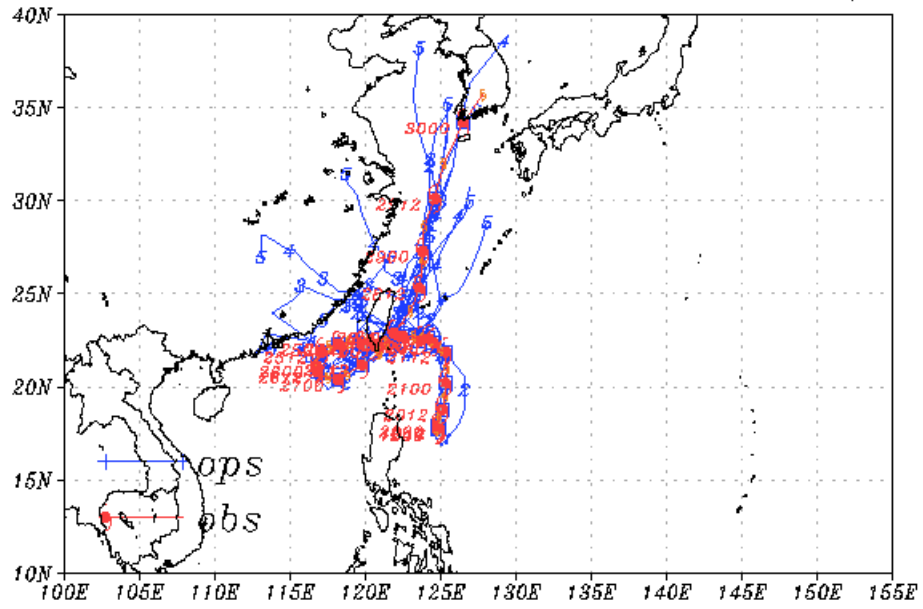
HAIKUI 2012:8:3:0-2012:8:9:0 (SLP)



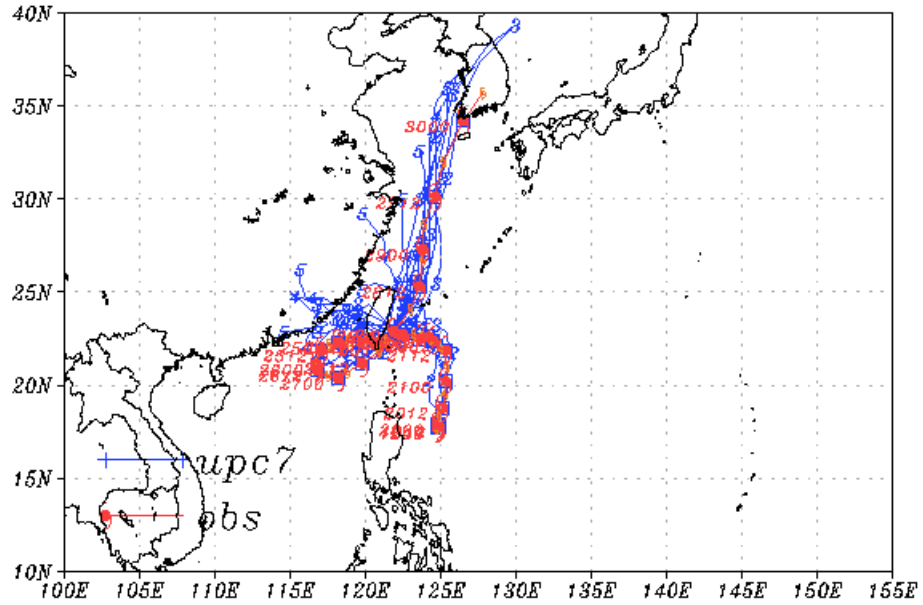
2012/5-2012/8 (10 cases) average typhoon track error



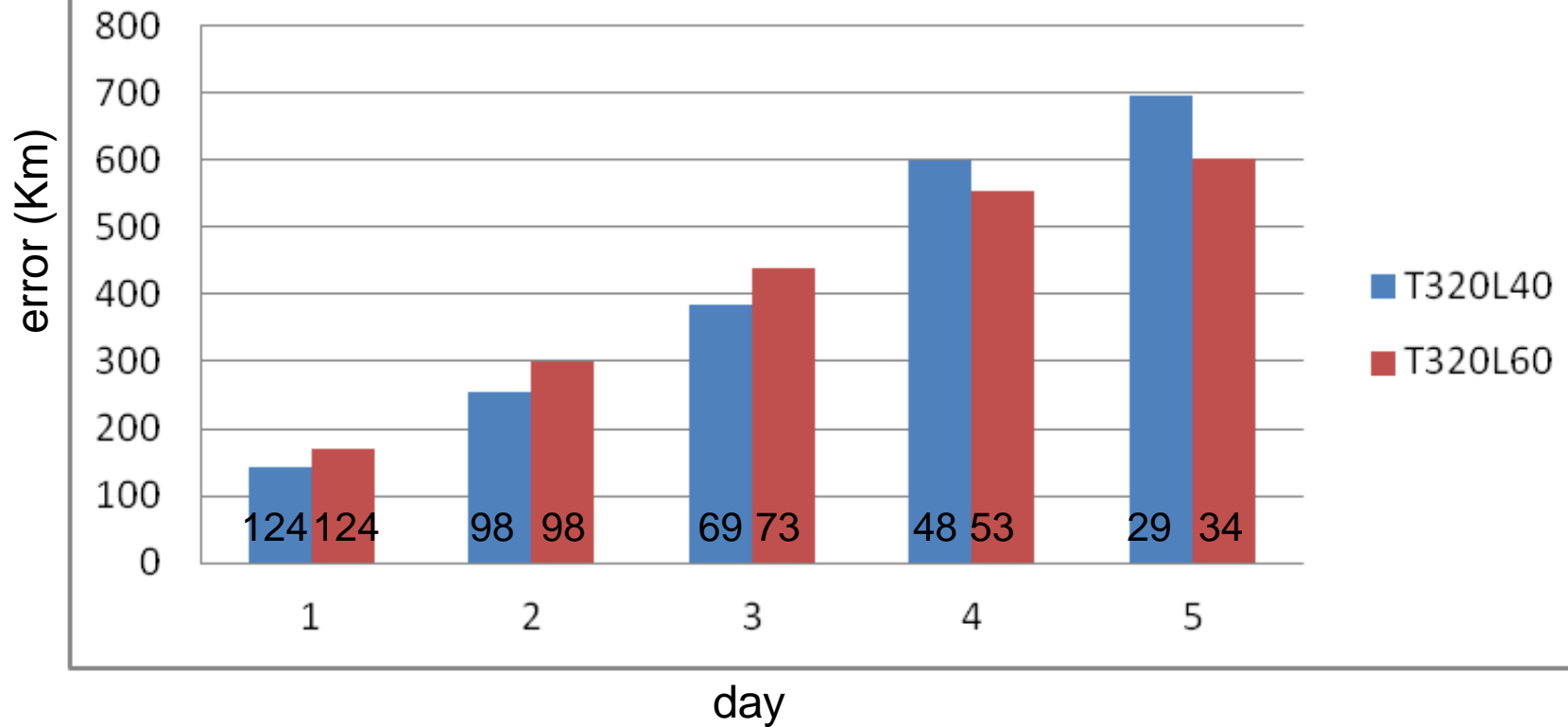
TEMBIN 2012:8:19:0-2012:8:30:0 (SLP)



TEMBIN 2012:8:19:0-2012:8:30:0 (SLP)



2012/5-2012/9 (13 cases) average typhoon track error



二、CWBGFS未來的發展

1. 分析模組

- a. 增加新衛星觀測資料同化 (2013?)
- b. Global hybrid data assimilation (2014?)

2. 預報模式

- a. T512L60 (25km, S-P坐標, model top=0.1mb) (2013)
- b. 物理參數化改進(2013)
- c. TL768L60 (semi-lagrangian) (2015?)

3. typhoon ensemble track prediction by using singular vector perturbation method (2015?)

增加新衛星觀測資料同化

觀測類別		內容
傳統觀測	地面觀測	SYNOP, SHIP, METAR, BUOY
	高空觀測	TEMP, PILOT, AIREP, PROFILER, NEXTRA
衛星觀測	衛星反演風	SATOB(GOES, GMS, METEOSAT, MTSAT), MODIS(AQUA, TERRA)
	Scaterometer	ASCAT, WINDSAT
	GPSRO	GPSRO(COSMIC, METOP-A/GRAS, GRACE-A, TERRASAR-X)
	SOUNDERS	AMSUA(NOAA15/18/19, METOP-A, AQUA), AMSUB/MHS(NOAA15/18/19/METOP-A), HIRS3/4(NOAA17/19, METOP), AIRS(AQUA), IASI(METOP-A), AMSRE(AQUA), SNDR(GOES11/12)
	IMAGERS	IMAGERS(GOES11/12)
	OZONE	OBSUV8(NOAA17/18), OMI(AURA)
	precipitable water	SSMI, TMI

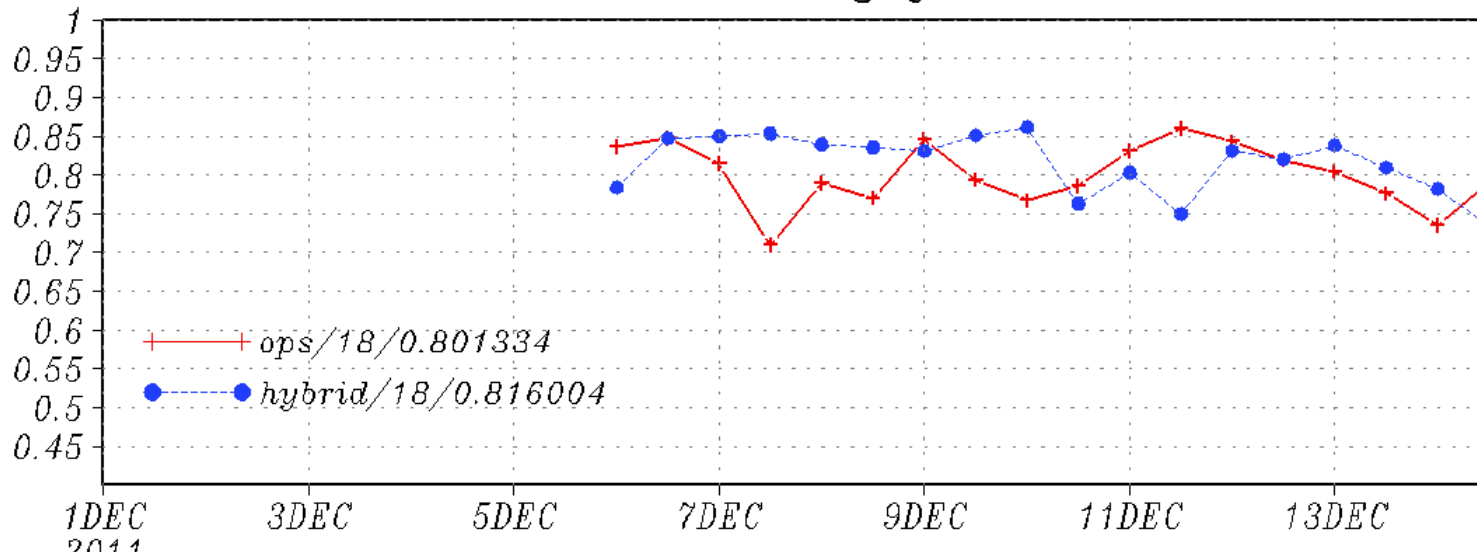
— used

— not used

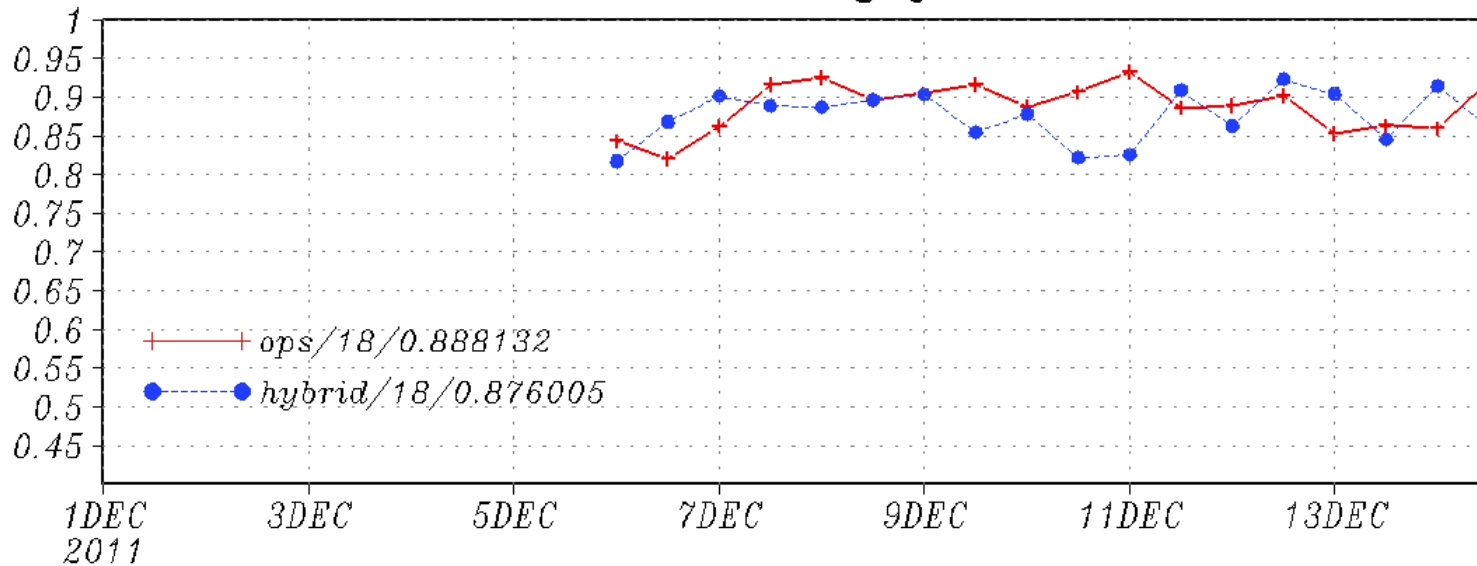
— Will be used

Global hybrid data assimilation

500 ach 5 day fcst - NA



500 ach 5 day fcst - SA



2. 預報模式

物理參數化

- a. **Noah 地表模式 (2013)**
- b. 地表通量參數化：參考Businger et al. (1971)。
- c. 大氣垂直紊流混合參數化：考慮非局部紊流通量之一階閉合邊界層參數法(Troen and Mahrt 1986)。
- d. 重力波拖曳參數化：依據Palmer et al. (1986)。
- e. 積雲對流參數化：Pan and Wu (1995) 的簡化A-S積雲對流參數法。
- f. 淺積雲對流參數化：採用Li (1994)、Li and Wang (2000)。
- g. 網格尺度降水參數化：採用Zhao and Frederick(1997) 預報雲水(雲冰)，並透過雲物理過程決定降水。
- h. 輻射參數化：短波及長波輻射模式均建立在二向式(two-stream)之計算結構，但長波方面使用Fu et al. (1997) 的二向\四向混合法以提高其精確性。關於大氣氣體吸收係數與雲光學特性的參數化乃參考Fu and Liou (1992; 1993)。
- i. **Nonorographic gravity wave drag parameterization (2013)**

國際各作業中心模式

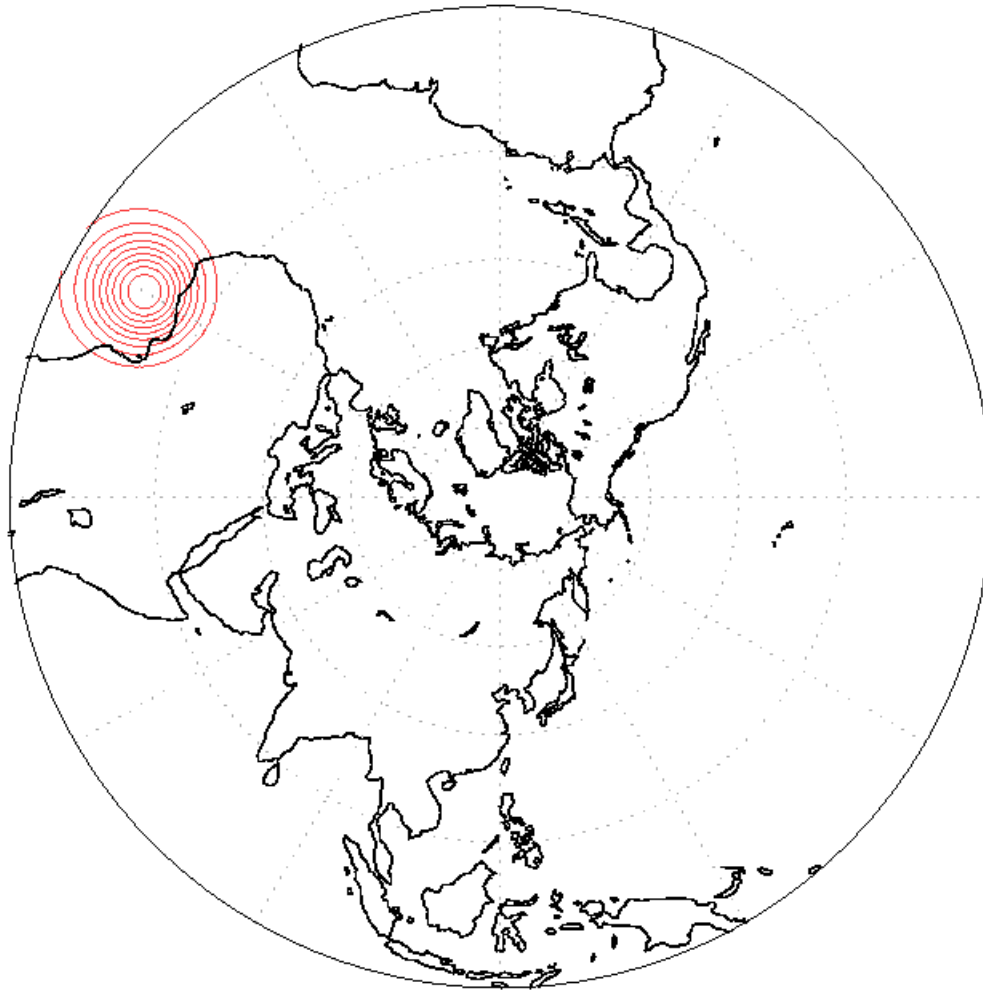
名稱	解析度	水平	垂直座標 層數	模式架構
ECMWF (2010/1)	TL1279L91	16km	H-B 91	Semi_L+spectrum
JMA (2007/11)	TL959L60	20km	H-B 60	Semi_L+spectrum
NCEP(2010/7)	T574L64	23km	H-B 64	spectrum
UKM (2010/5)	N512L70	25km	H-B 70	Semi_L+Finite difference
KMA (韓國2010/5)	N512L70	25km	H-B 70	Semi_L+Finite difference
CWB (2013/5)	T512L60	25km	H-B 60	spectrum
CMA (中國2008/1)	TL639L60	32km	H-B 60	Semi_L+spectrum
GEM (加拿大2006/8)	(800X600)	34km	H-B 80	Semi_L+finite element
NOGAPS (2009/9)	T319L42	42km	H-B 42	spectrum

Semi-lagrangian semi-implicit model development

1. Scheme Tools
2. Shallow water test
3. Semi-implicit method

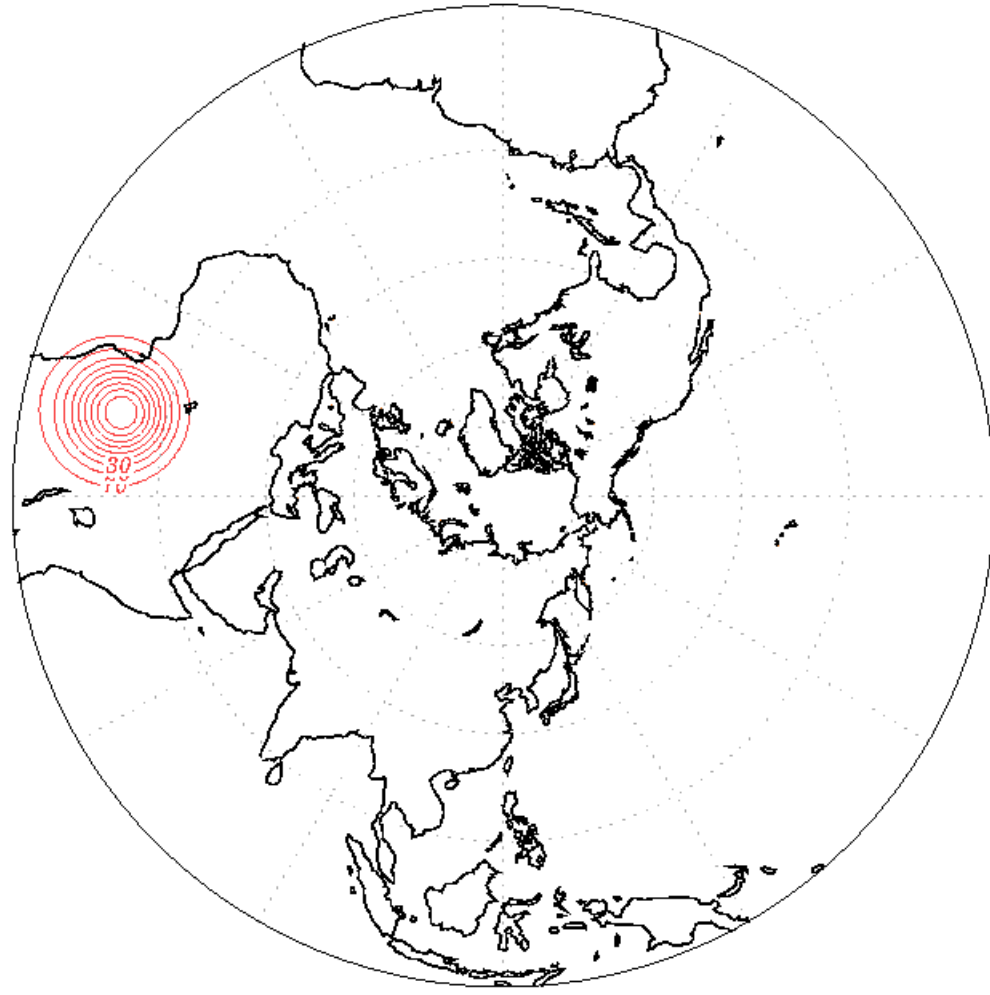
NORTH POLE

h500 analysis (DATA_solid)



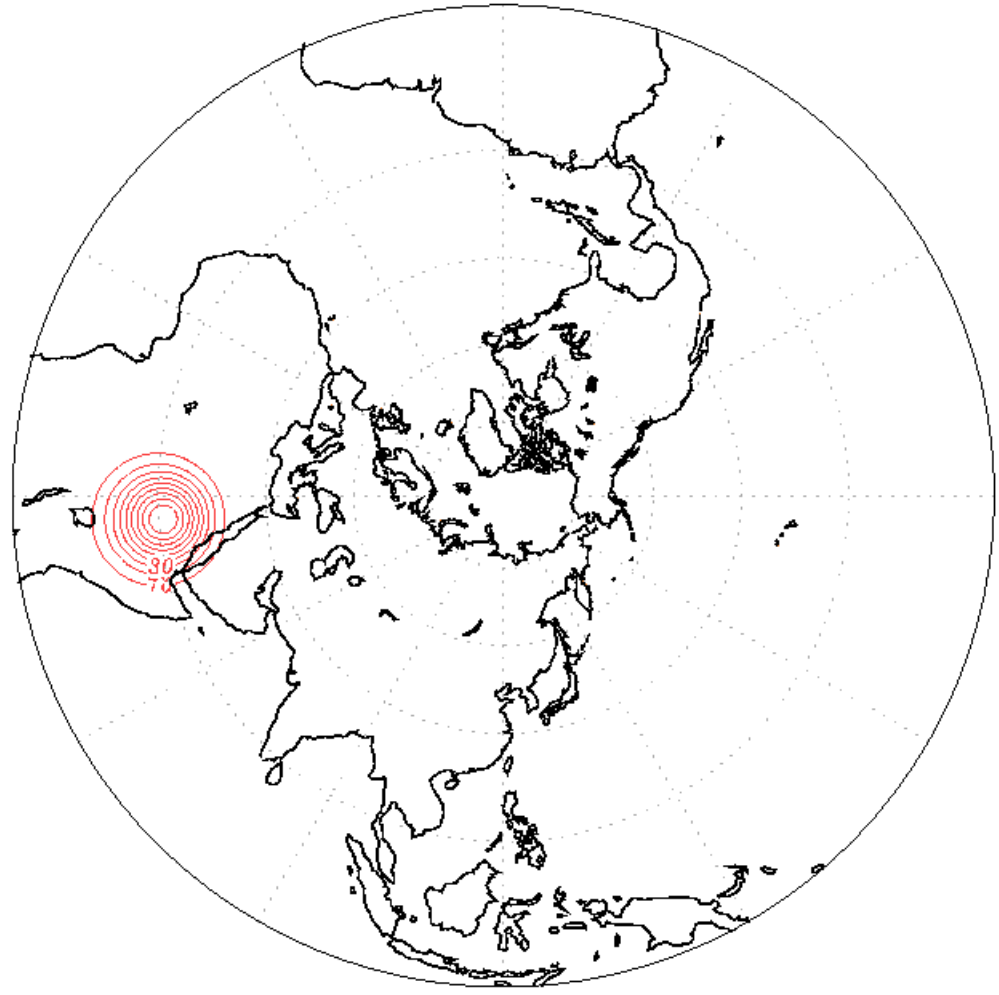
plot interval=10 with factor=1

h500 48hrs fcst (DATA_solid)



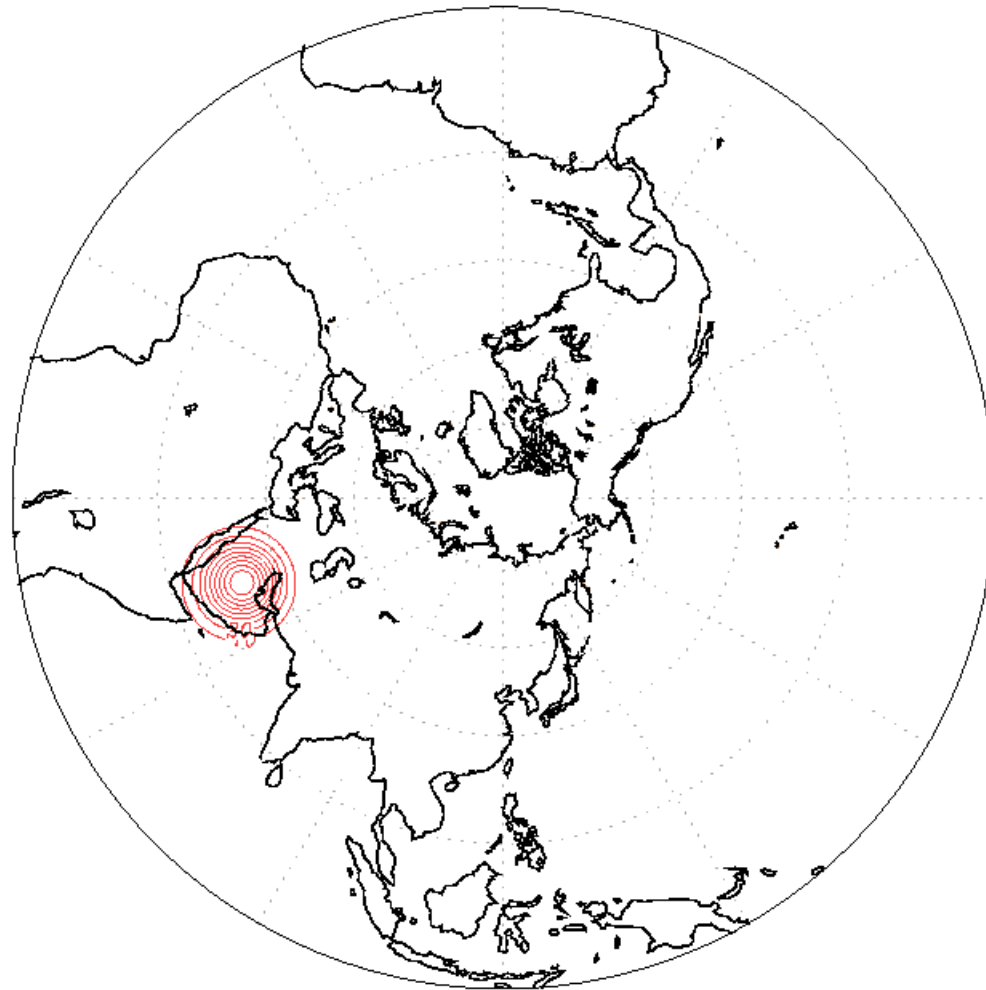
plot interval=10 with factor=1

h500 96hrs fcst (DATA_solid)



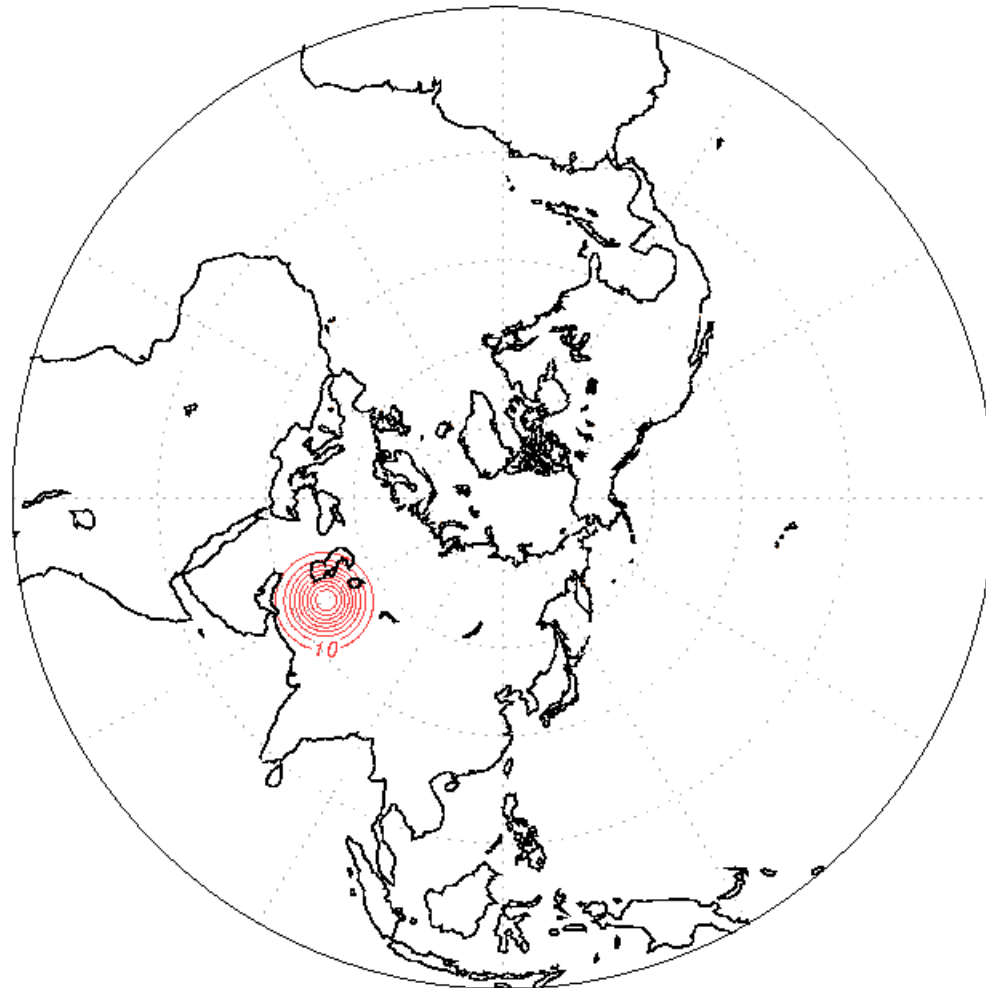
plot interval=10 with factor=1

h500 144hrs fcst (DATA_solid)



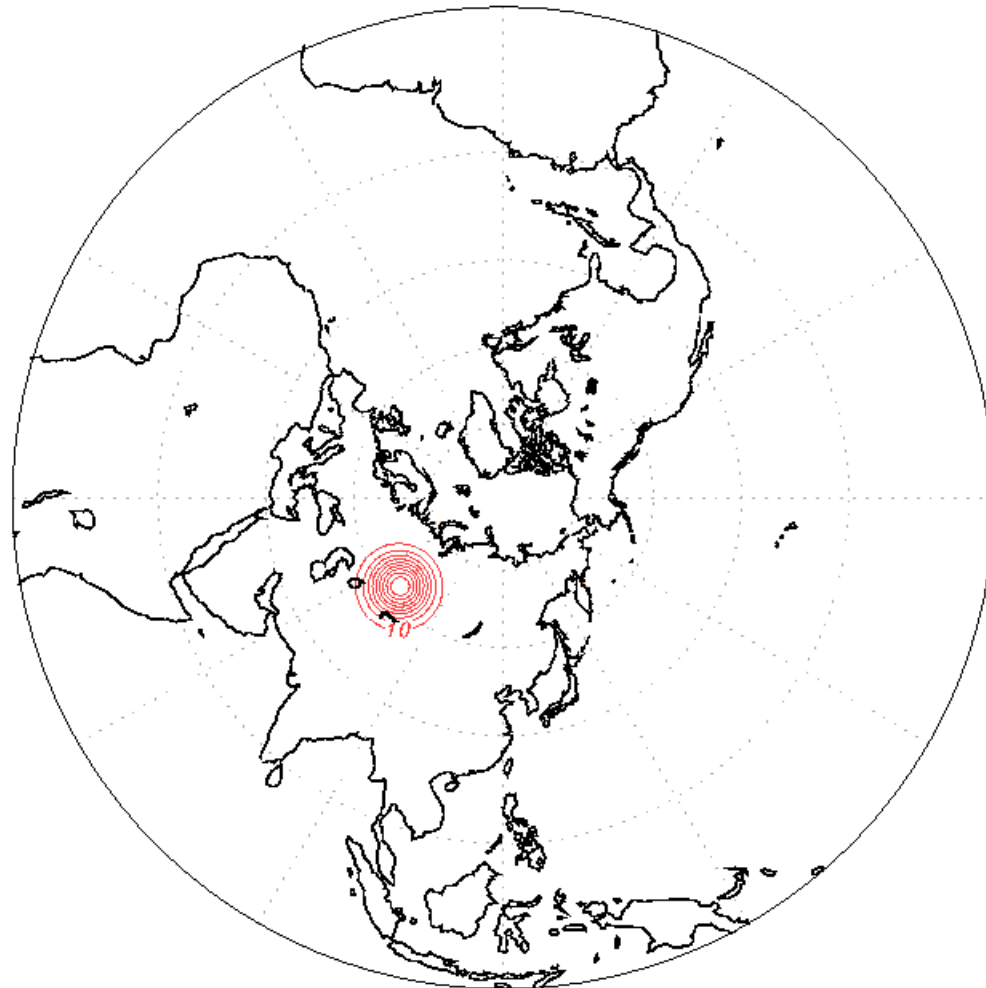
plot interval=10 with factor=1

h500 192hrs fcst (DATA_solid)



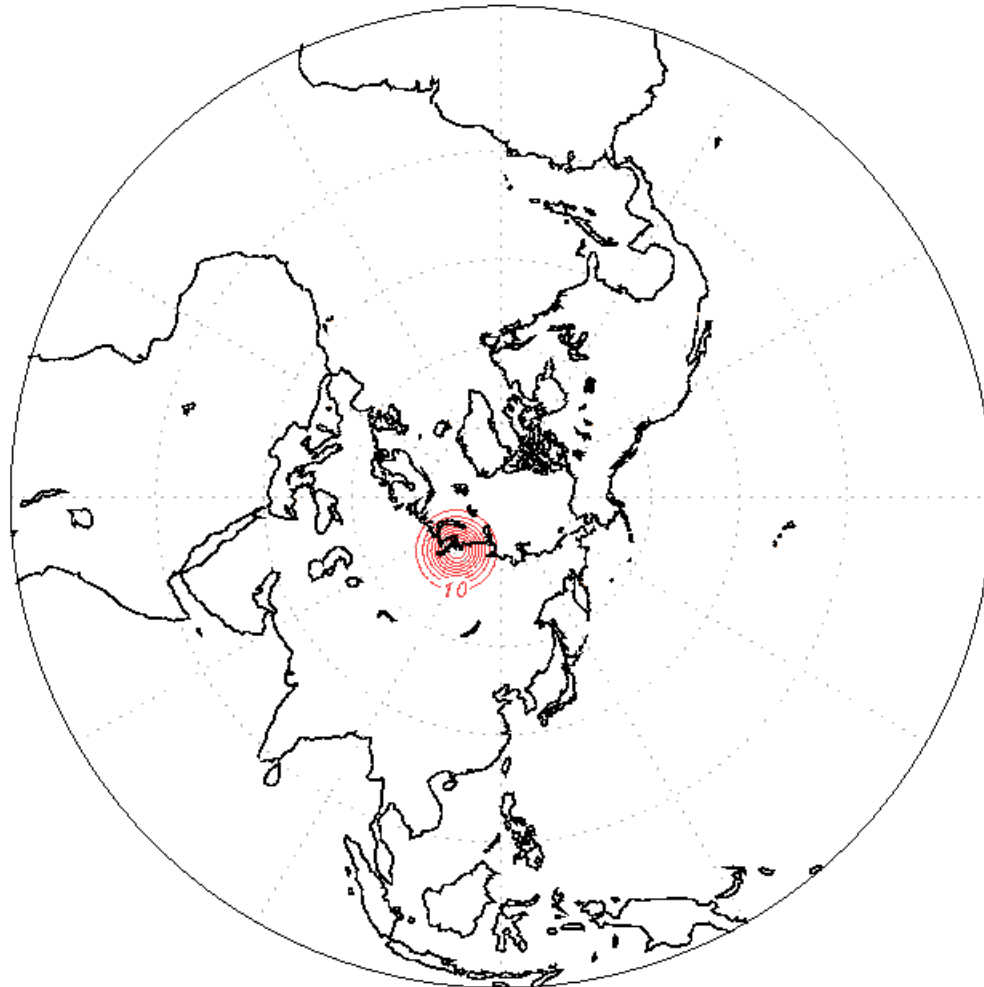
plot interval=10 with factor=1

h500 240hrs fcst (DATA_solid)



plot interval=10 with factor=1

h500 288hrs fcst (DATA_solid)



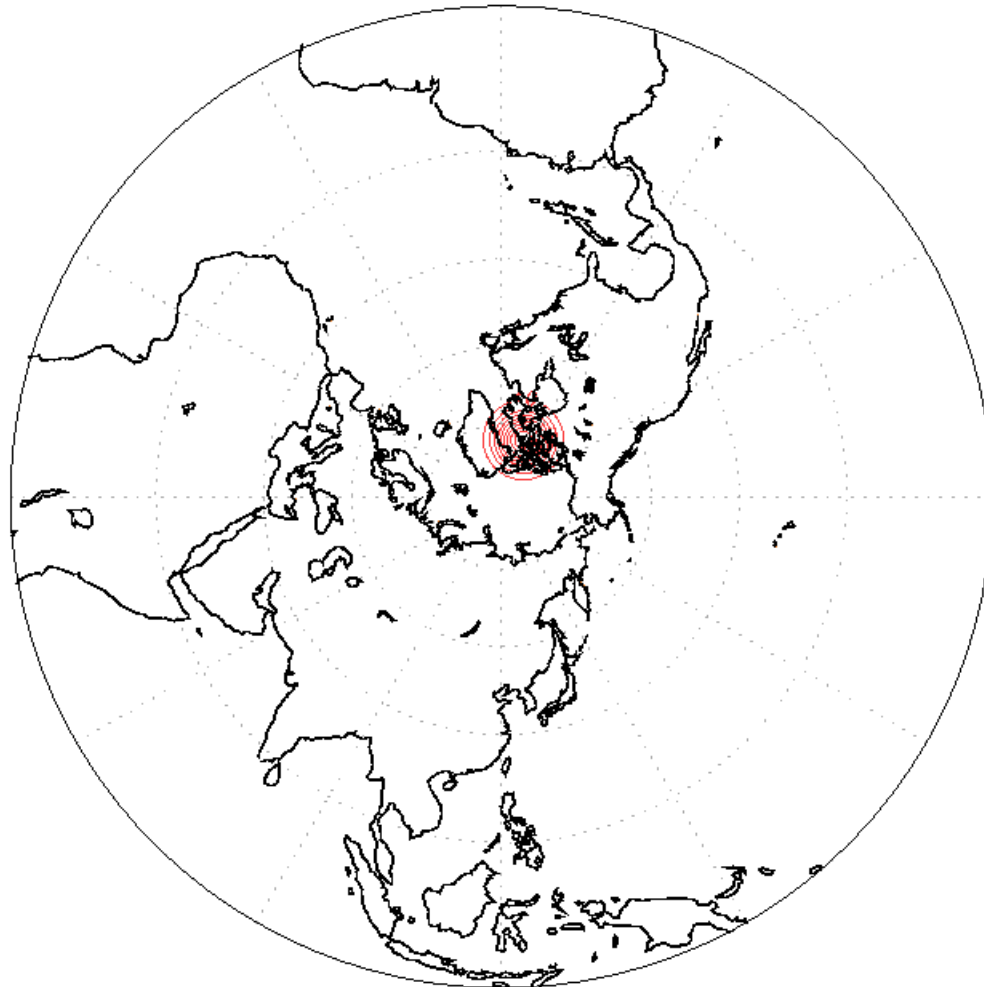
plot interval=10 with factor=1

h500 336hrs fcst (DATA_solid)



plot interval=10 with factor=1

h500 384hrs fcst (DATA_solid)



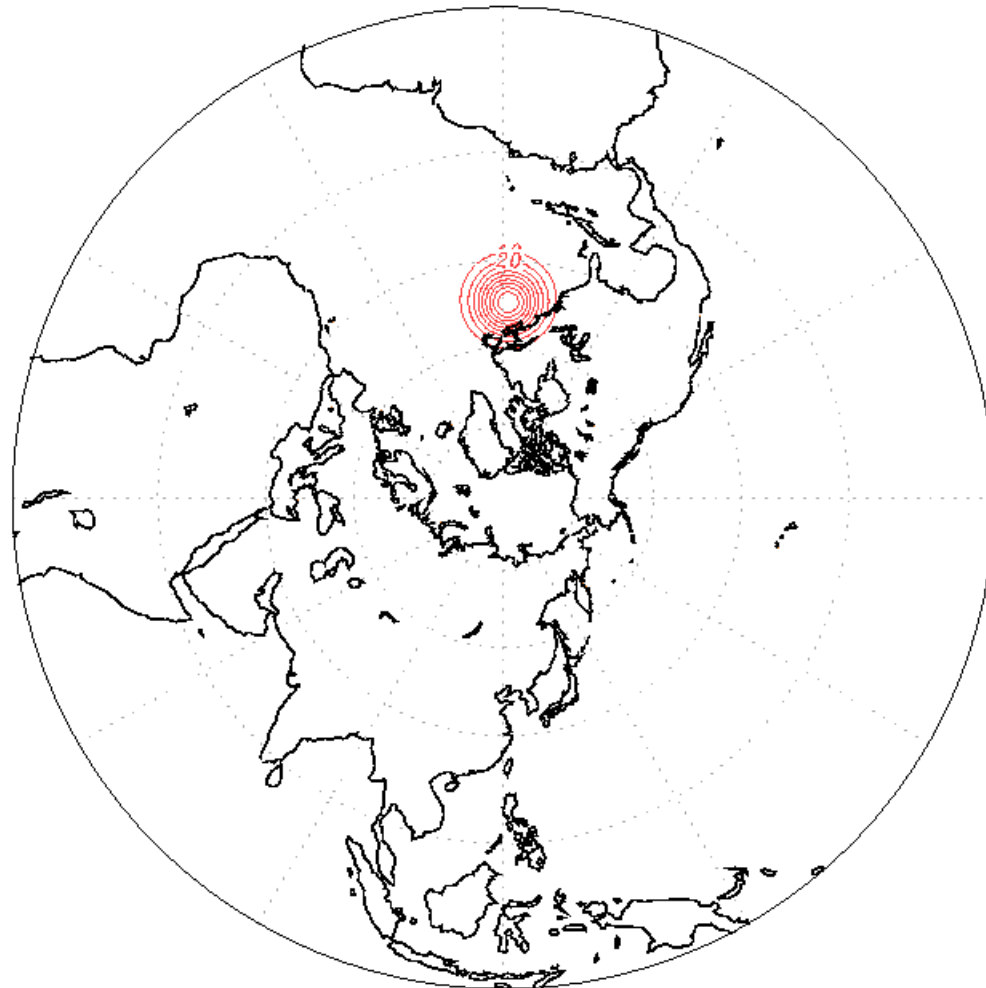
plot interval=10 with factor=1

h500 432hrs fcst (DATA_solid)



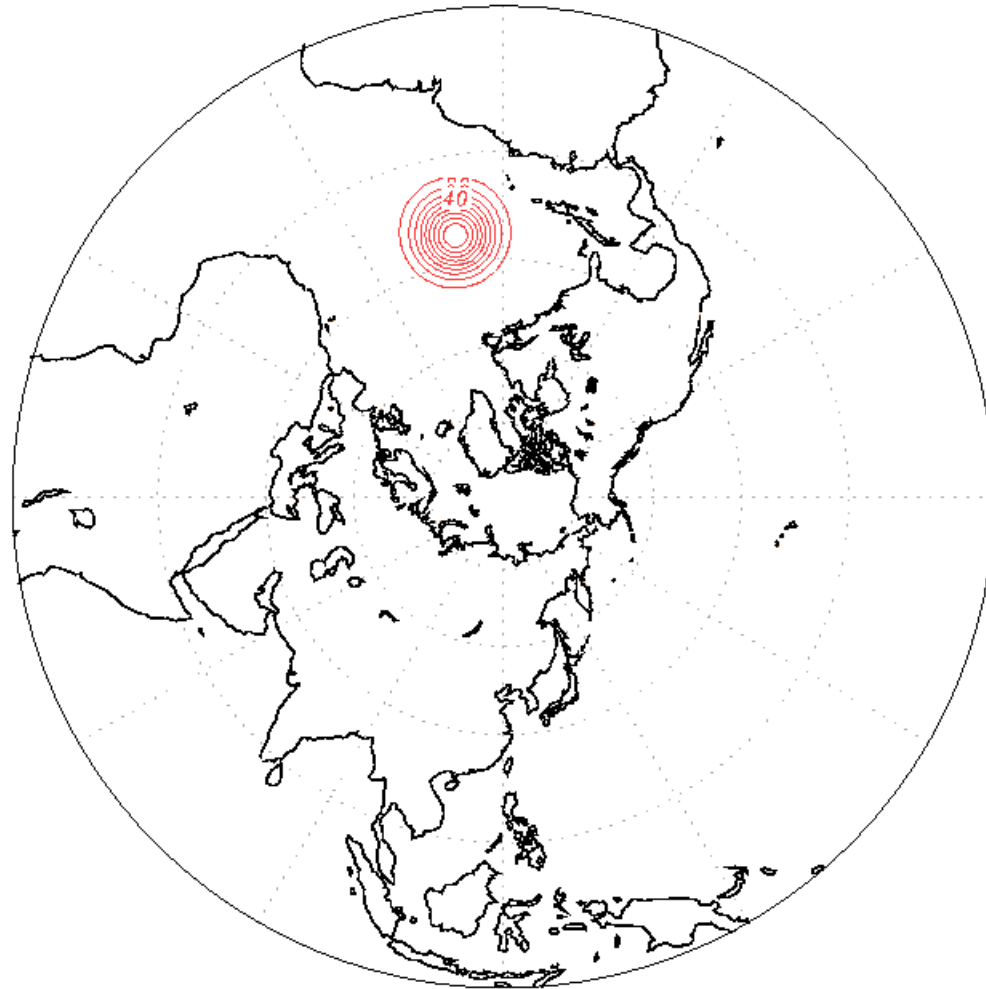
plot interval=10 with factor=1

h500 480hrs fcst (DATA_solid)



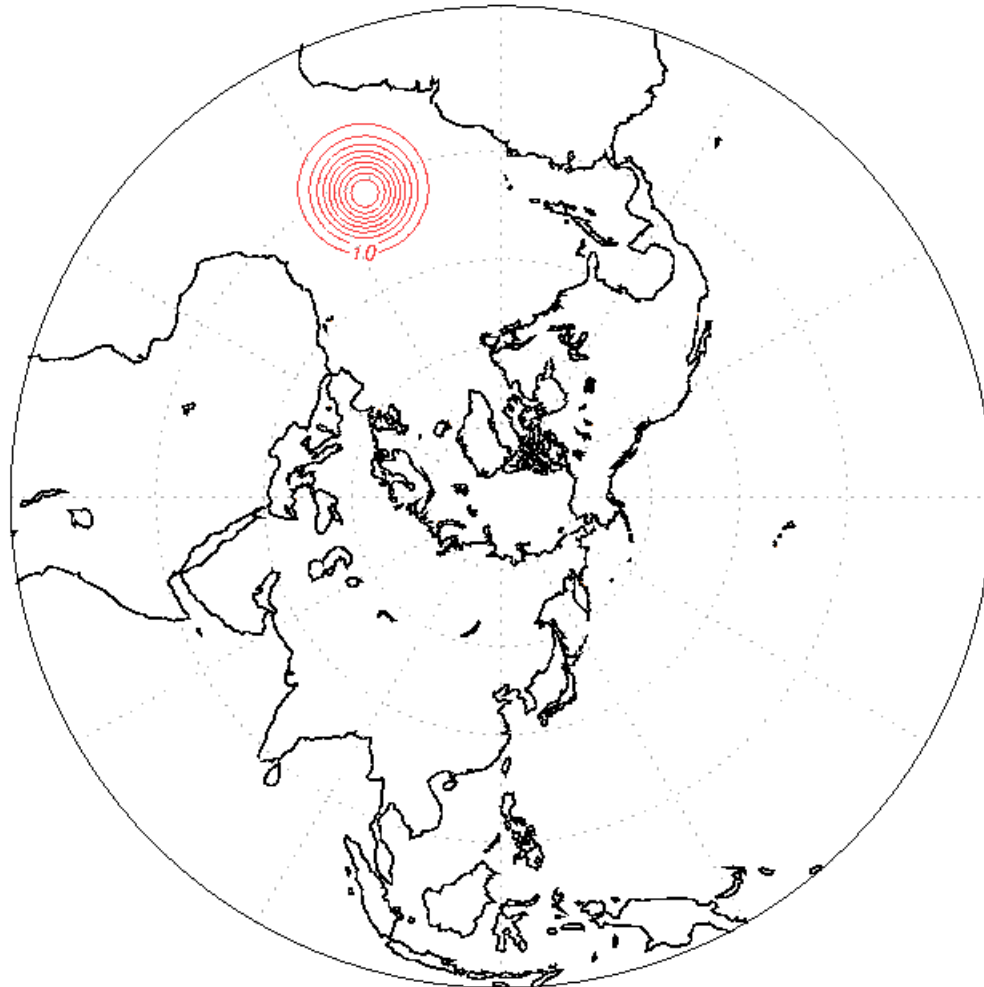
plot interval=10 with factor=1

h500 528hrs fcst (DATA_solid)



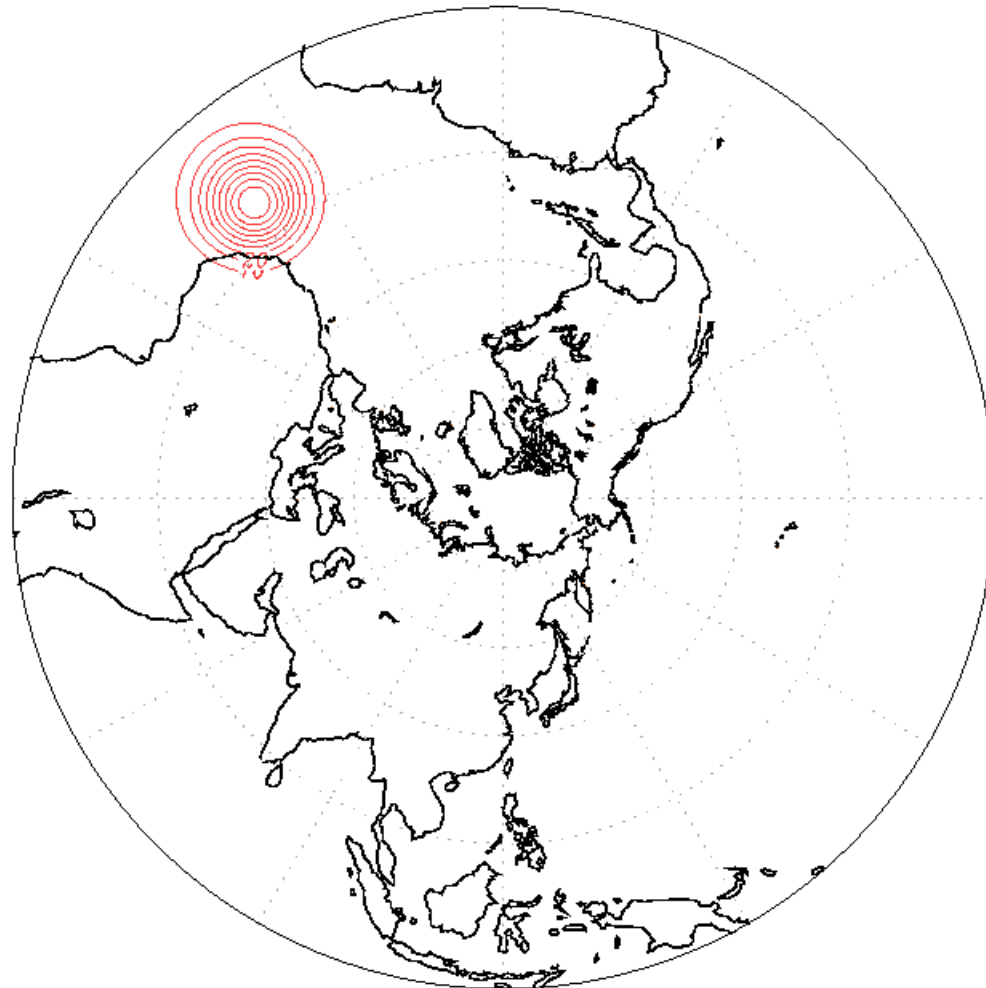
plot interval=10 with factor=1

h500 576hrs fcst (DATA_solid)



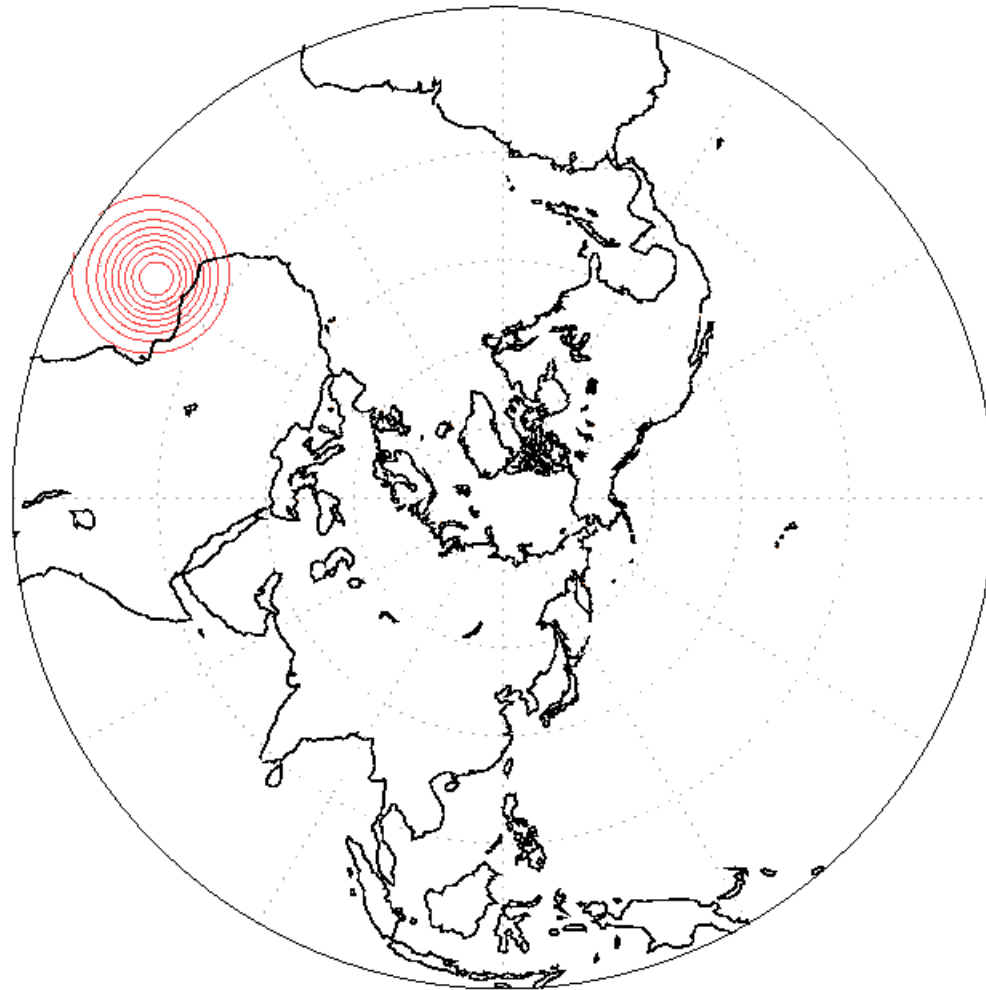
plot interval=10 with factor=1

h500 624hrs fcst (DATA_solid)



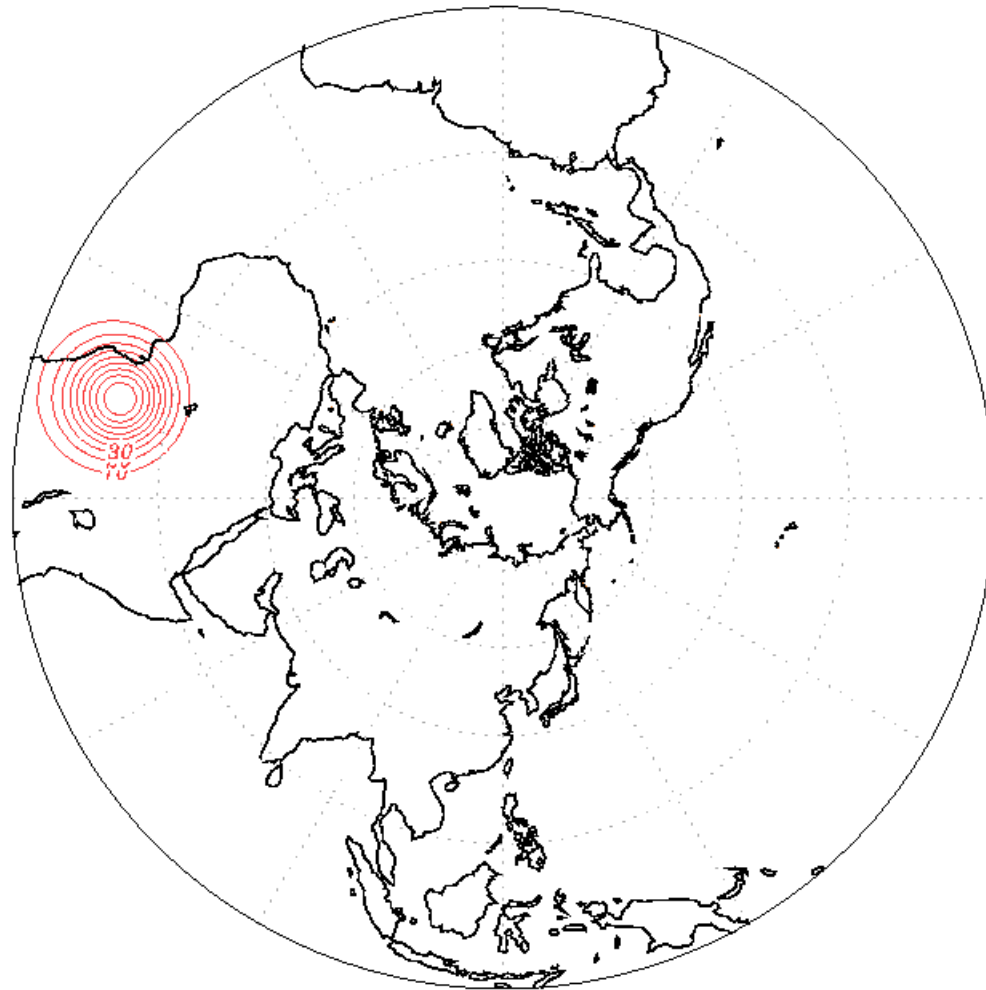
plot interval=10 with factor=1

h500 672hrs fcst (DATA_solid)



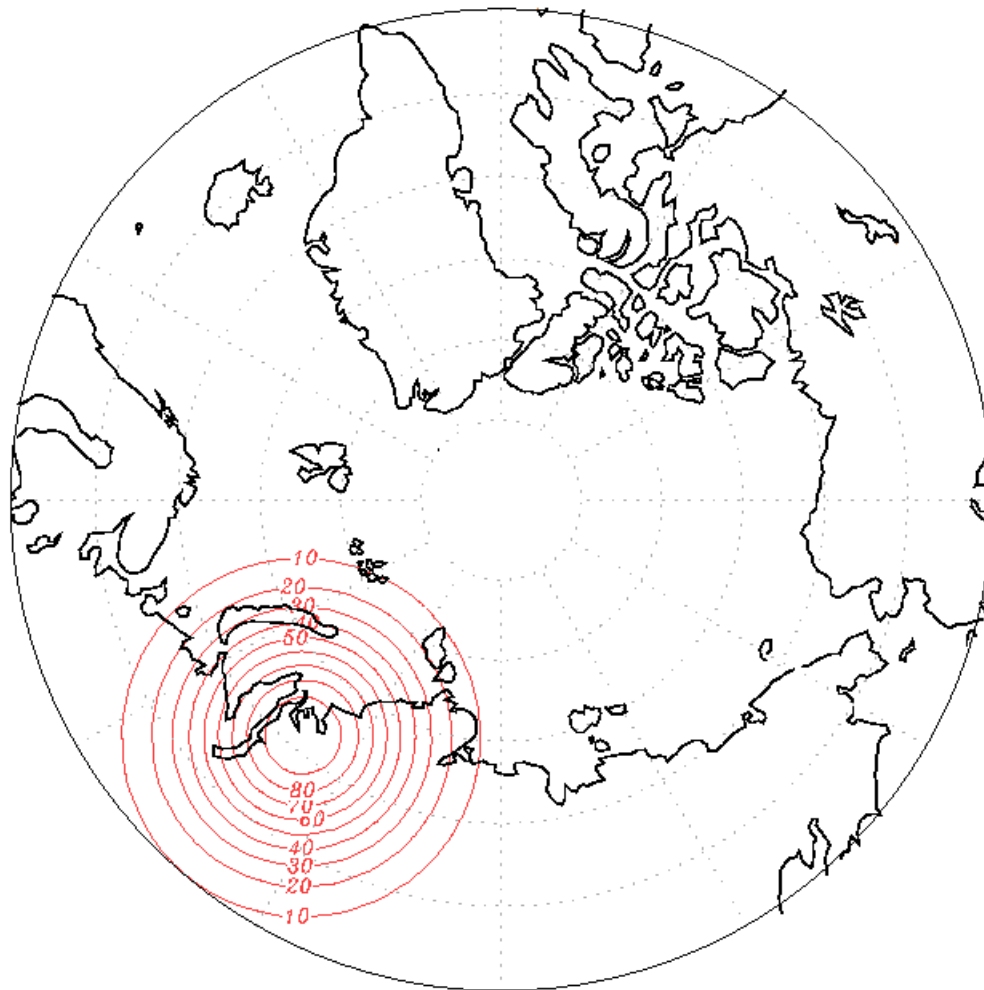
plot interval=10 with factor=1

h500 720hrs fcst (DATA_solid)



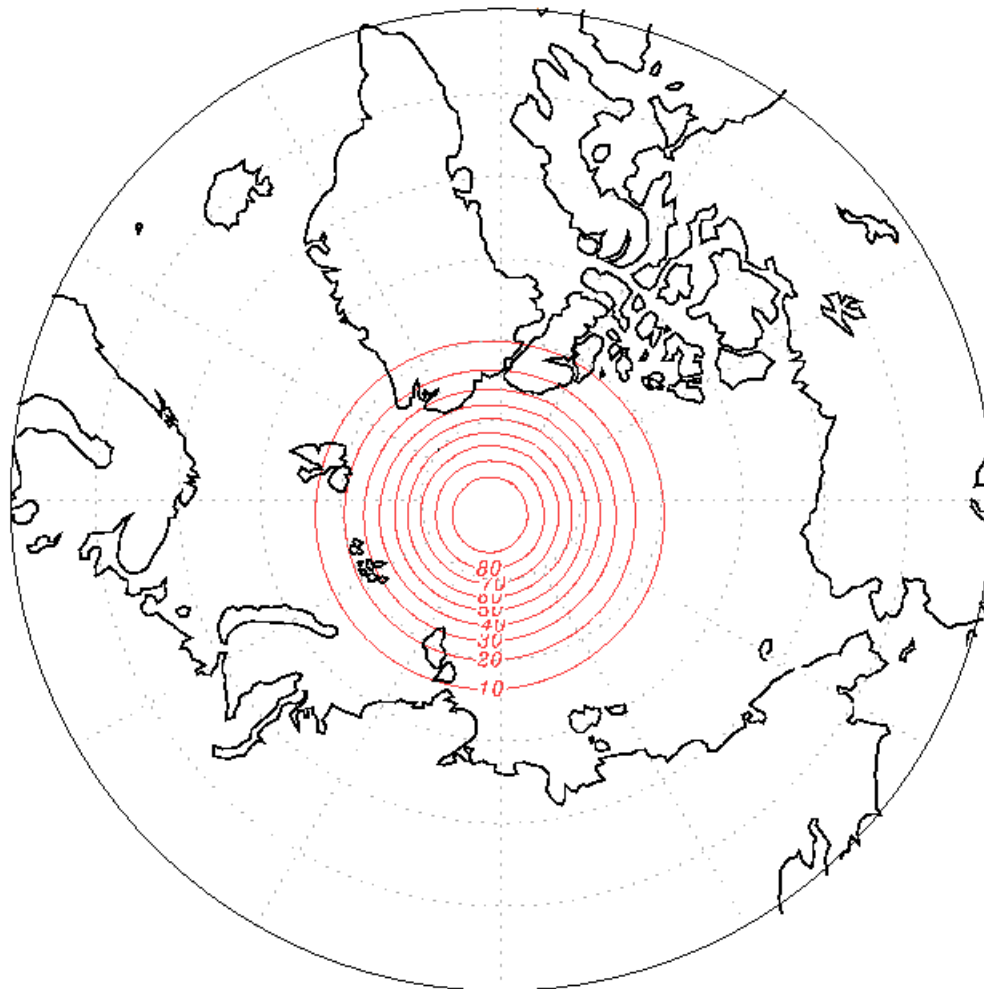
plot interval=10 with factor=1

h500 288hrs fcst (DATA_solid)



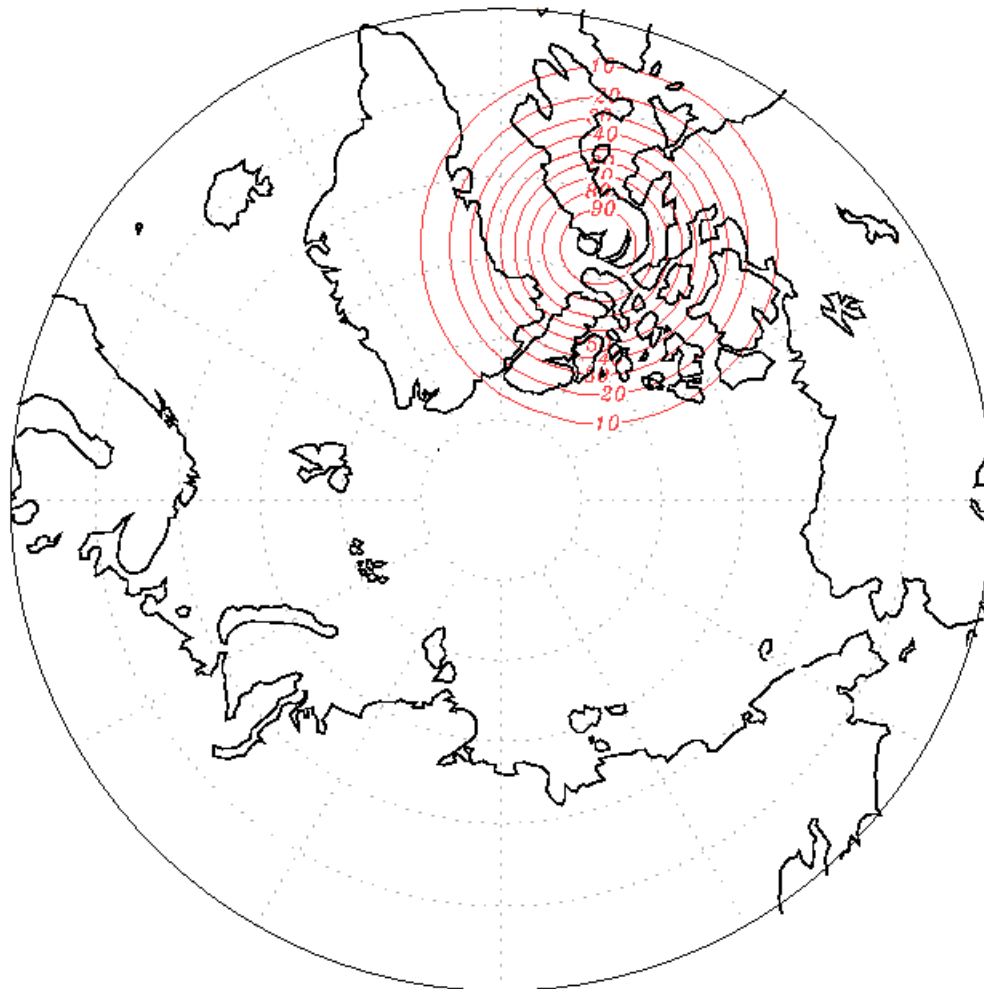
plot interval=10 with factor=1

h500 336hrs fcst (DATA_solid)



plot interval=10 with factor=1

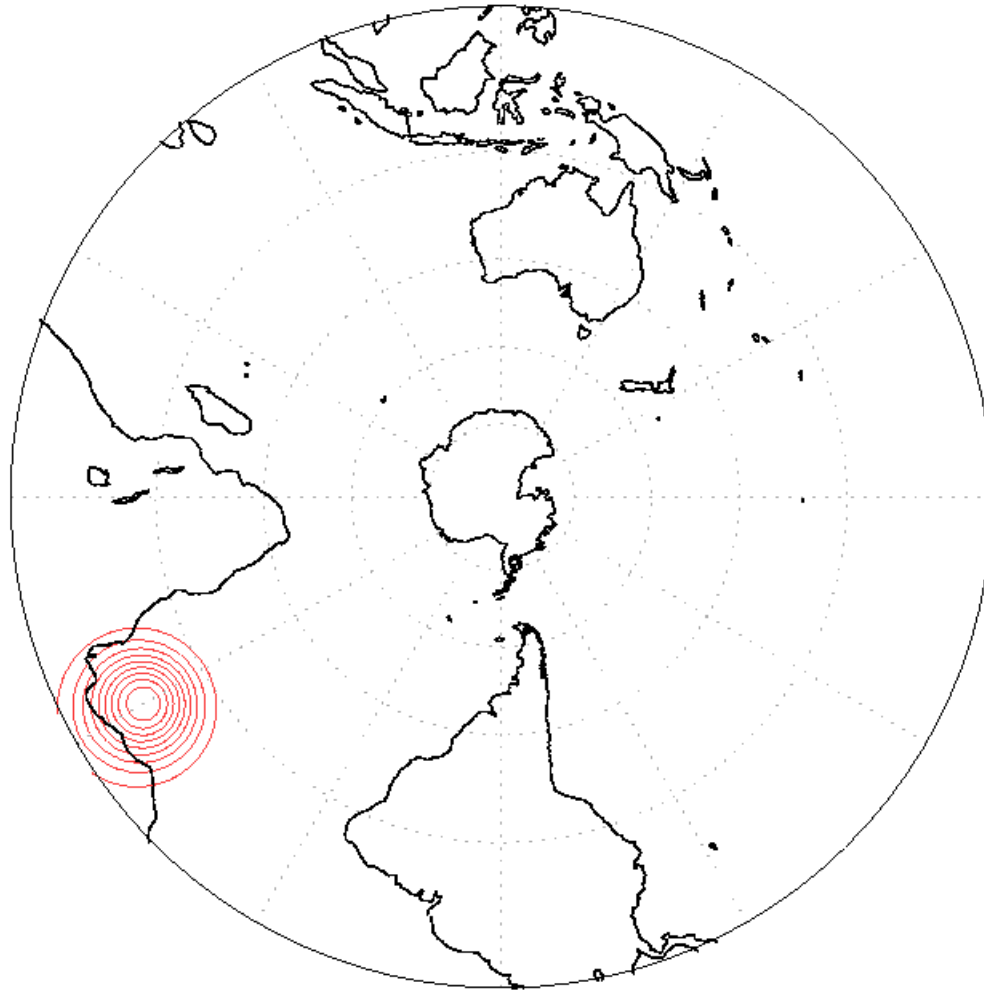
h500 384hrs fcst (DATA_solid)



plot interval=10 with factor=1

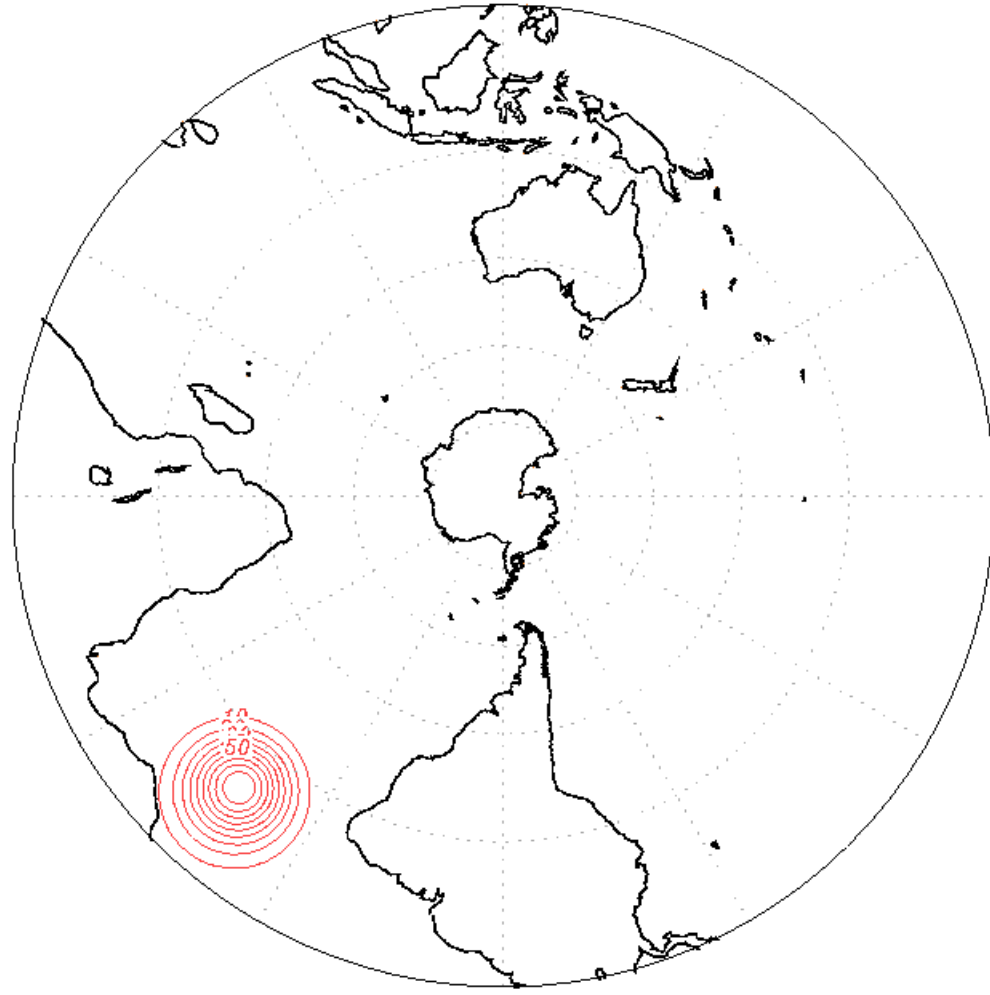
SOURTH POLE

h500 analysis (DATA_solid)



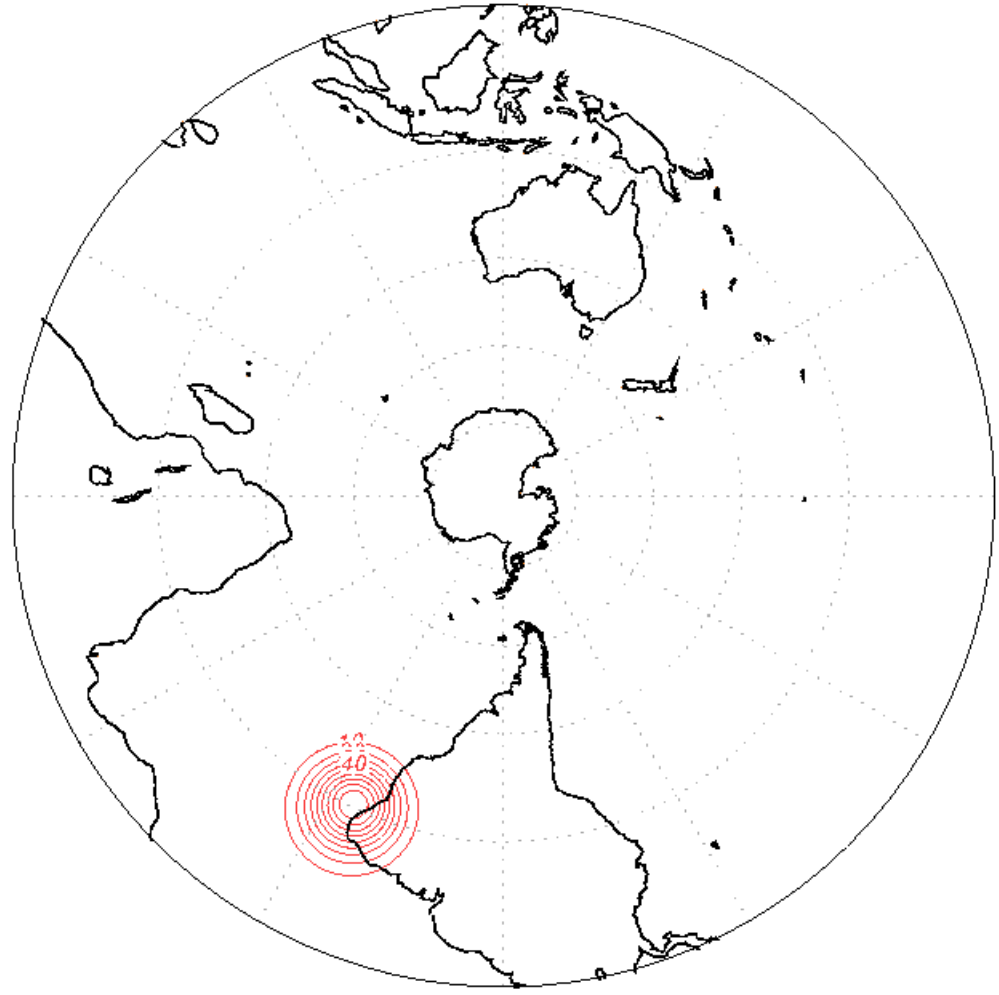
plot interval=10 with factor=1

h500 48hrs fcst (DATA_solid)



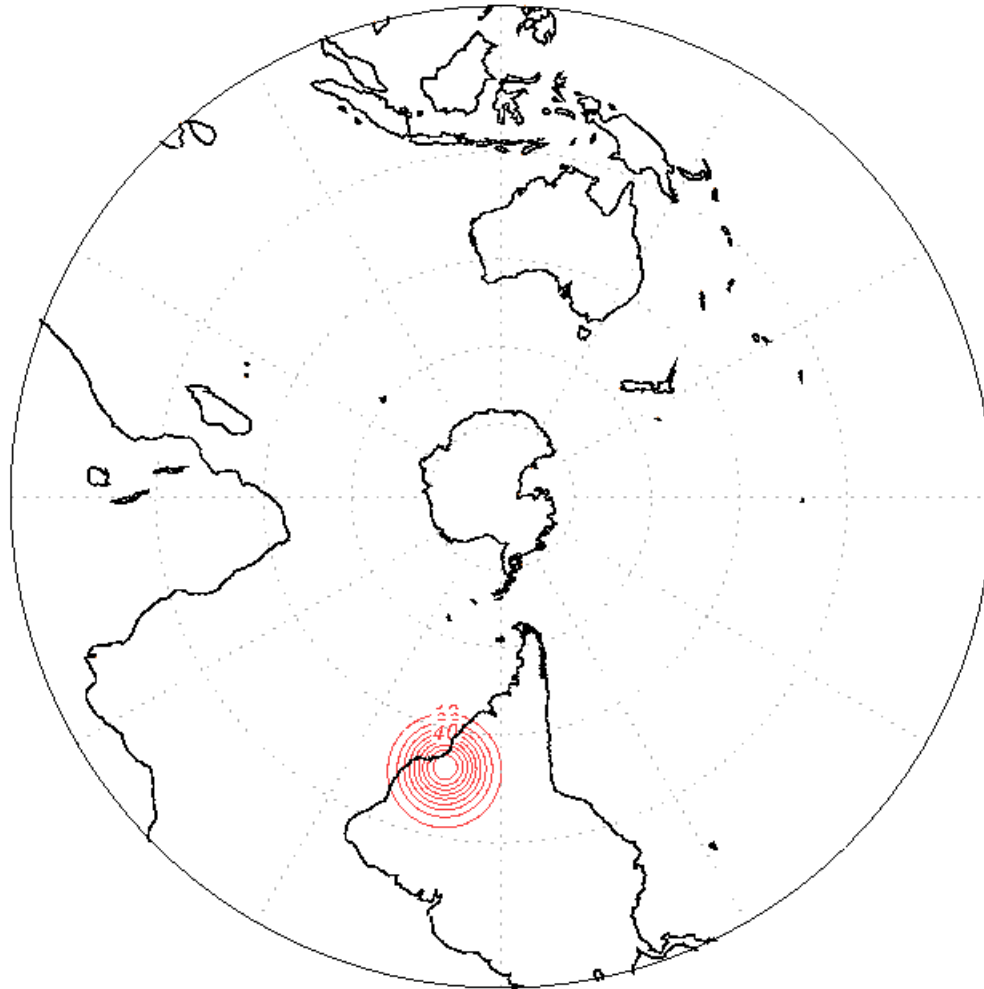
plot interval=10 with factor=1

h500 96hrs fcst (DATA_solid)



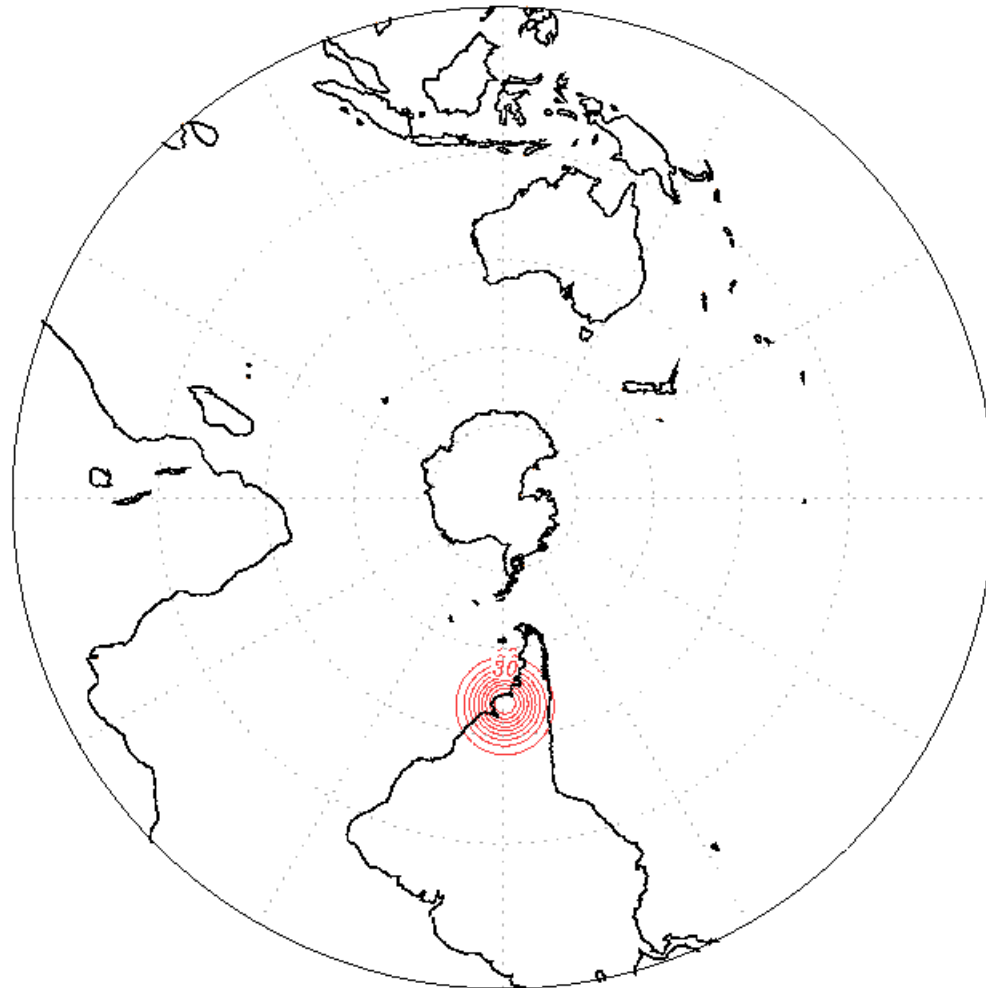
plot interval=10 with factor=1

h500 144hrs fcst (DATA_solid)



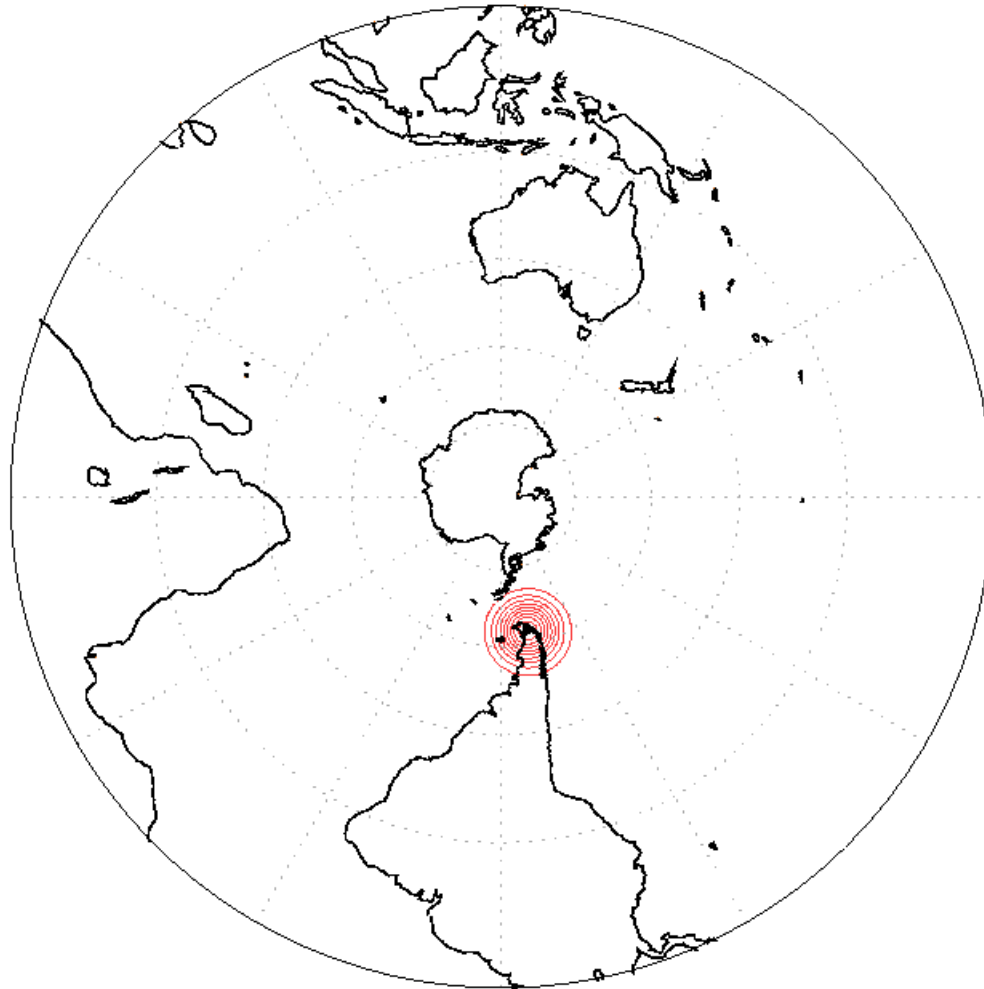
plot interval=10 with factor=1

h500 192hrs fcst (DATA_solid)



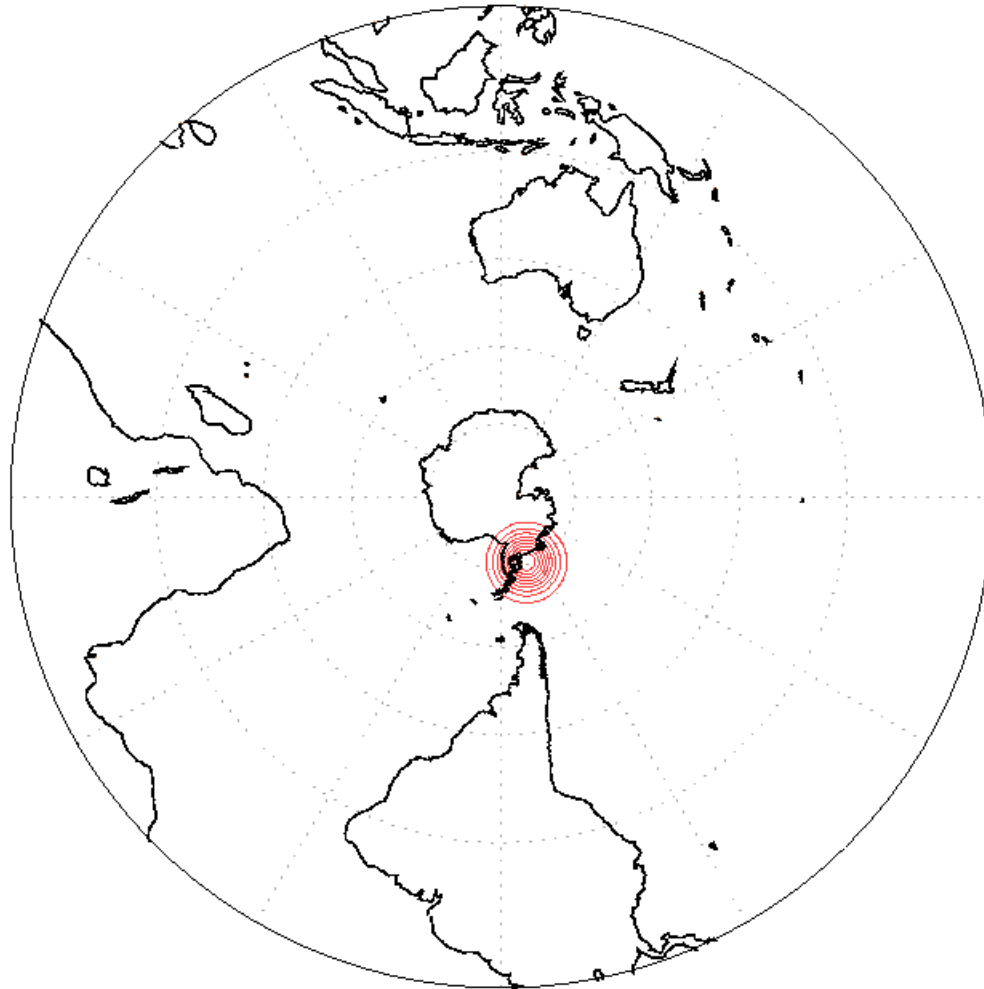
plot interval=10 with factor=1

h500 240hrs fcst (DATA_solid)



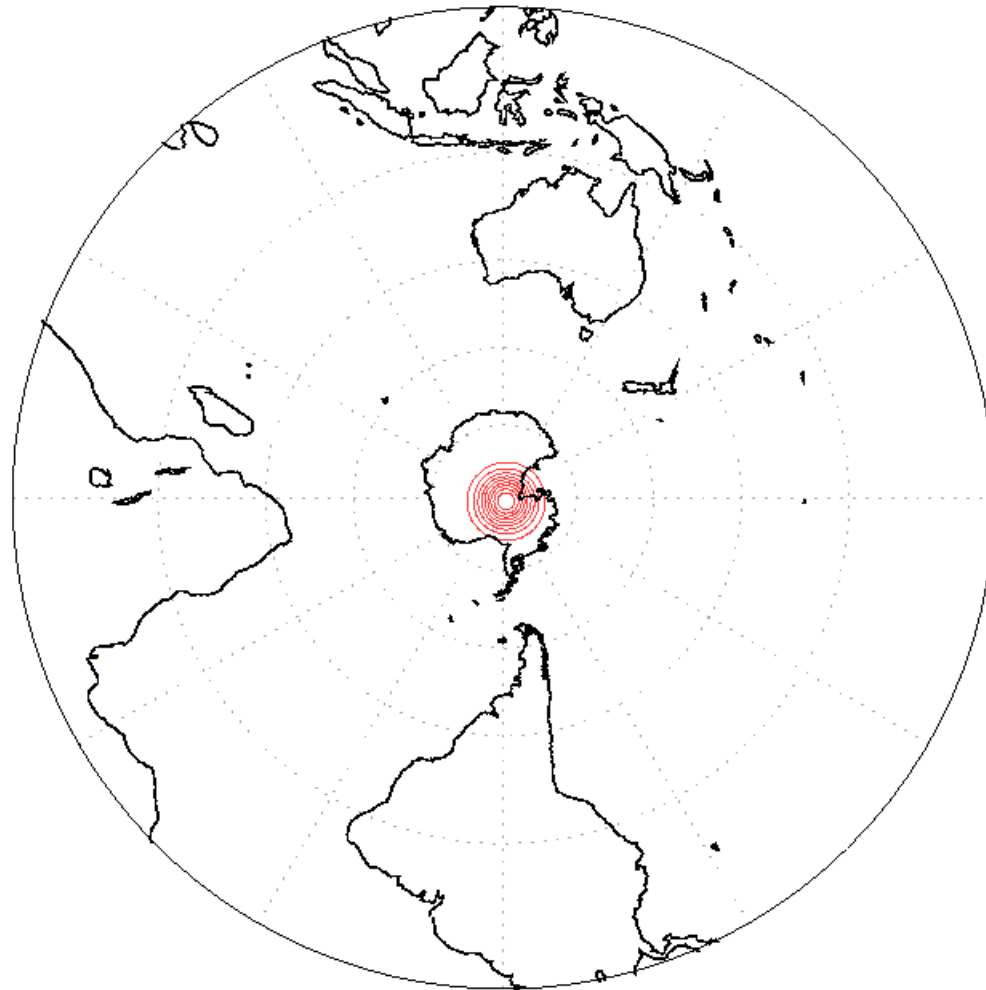
plot interval=10 with factor=1

h500 288hrs fcst (DATA_solid)



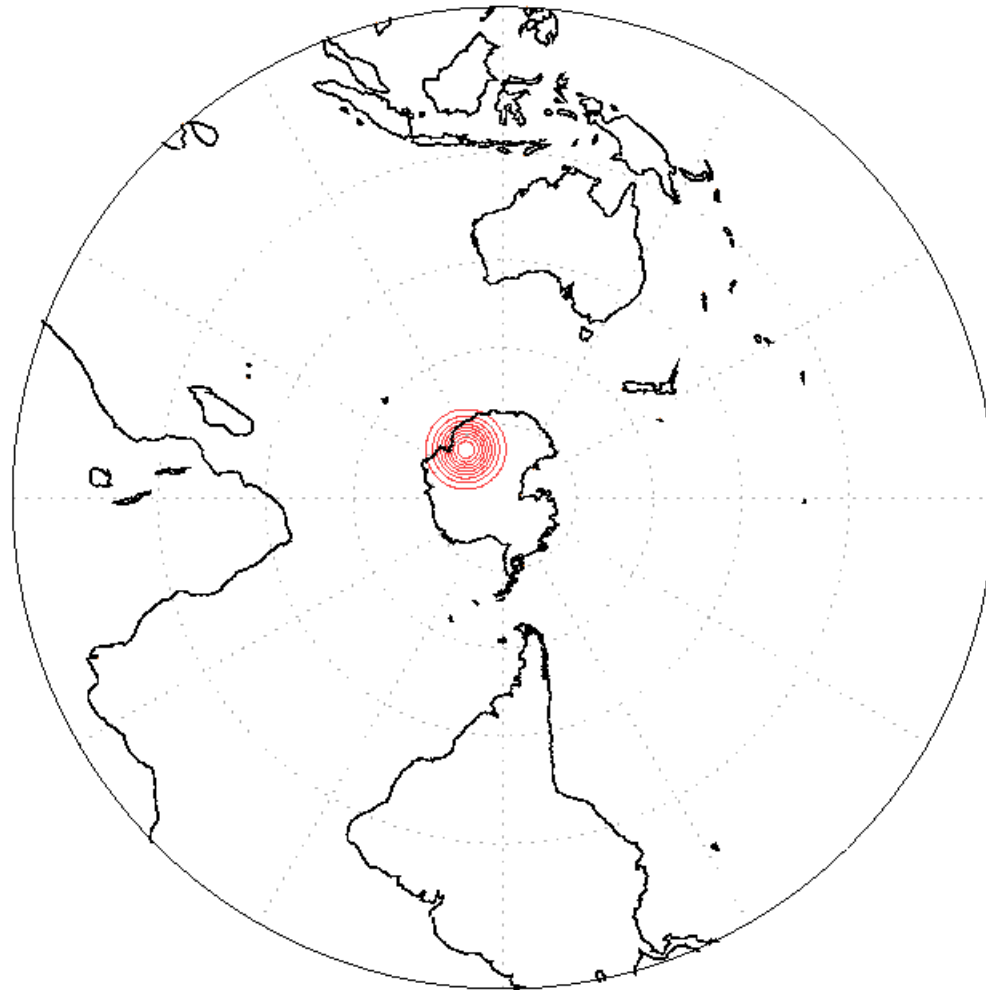
plot interval=10 with factor=1

h500 336hrs fcst (DATA_solid)



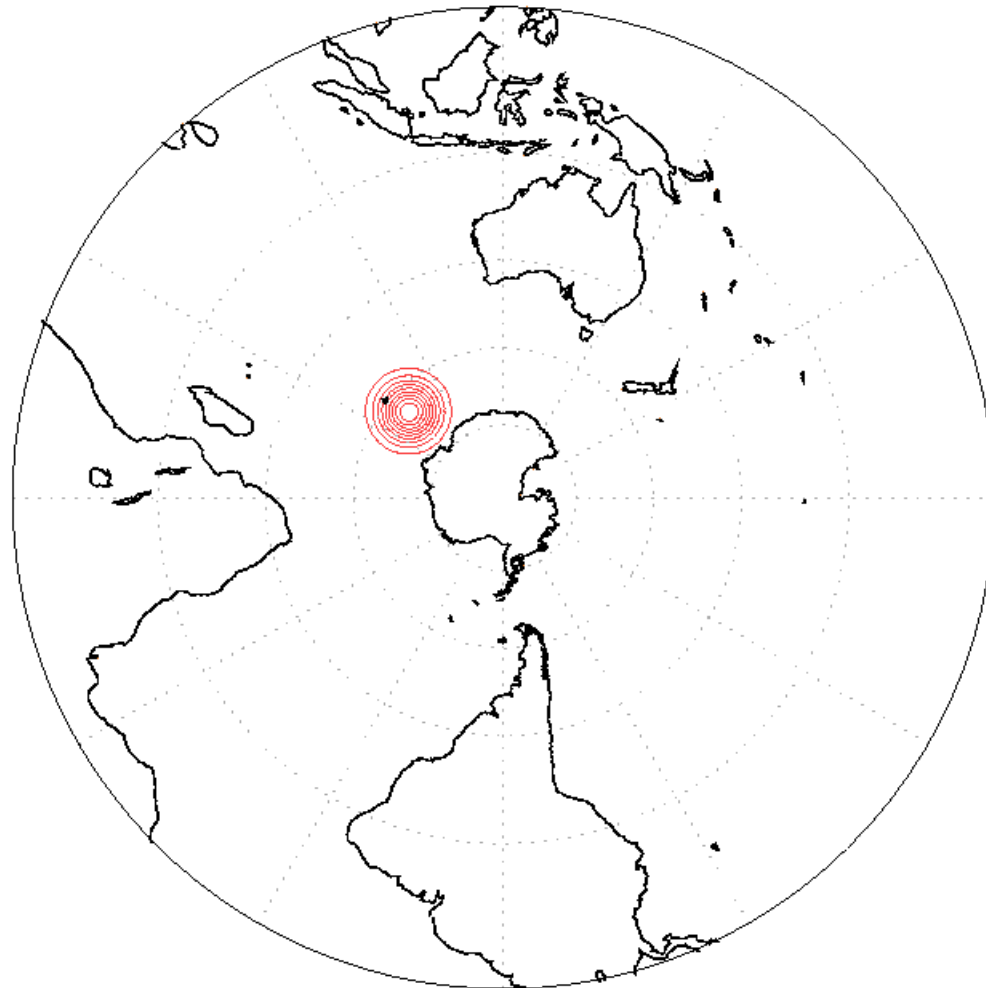
plot interval=10 with factor=1

h500 384hrs fcst (DATA_solid)



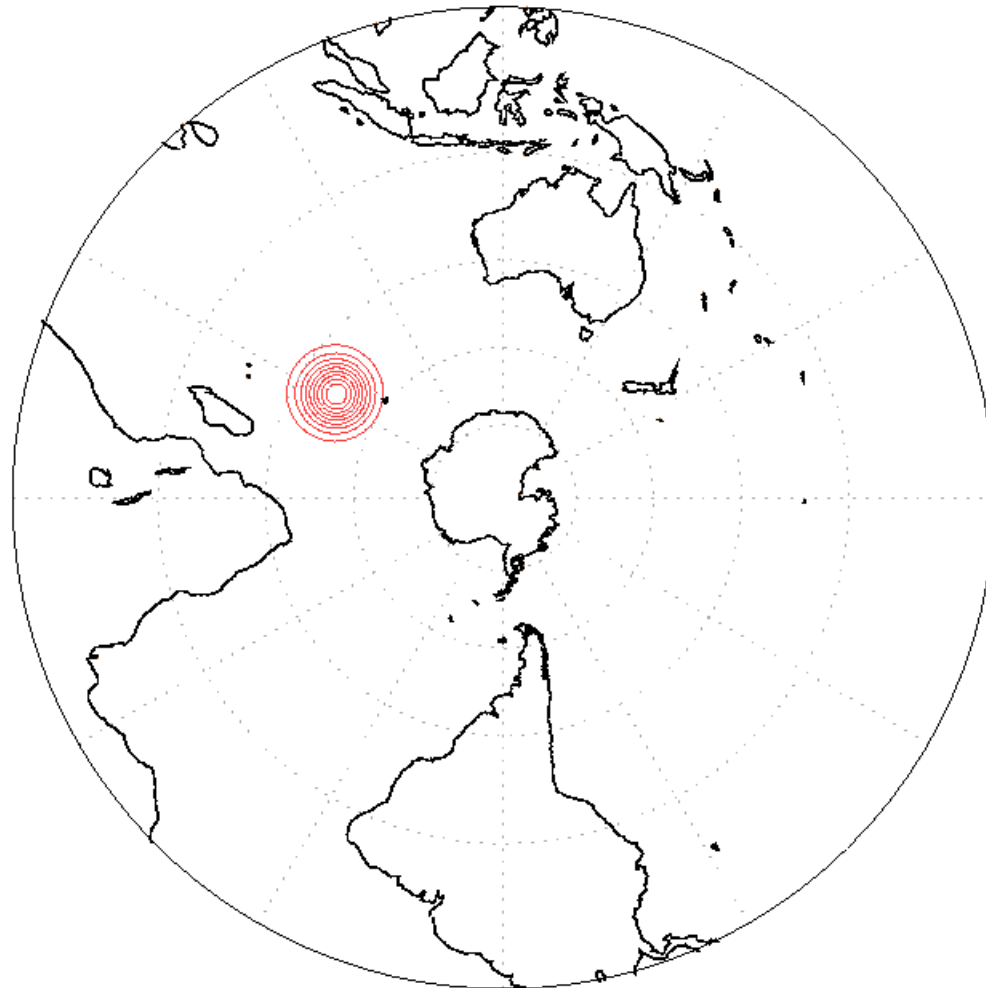
plot interval=10 with factor=1

h500 432hrs fcst (DATA_solid)



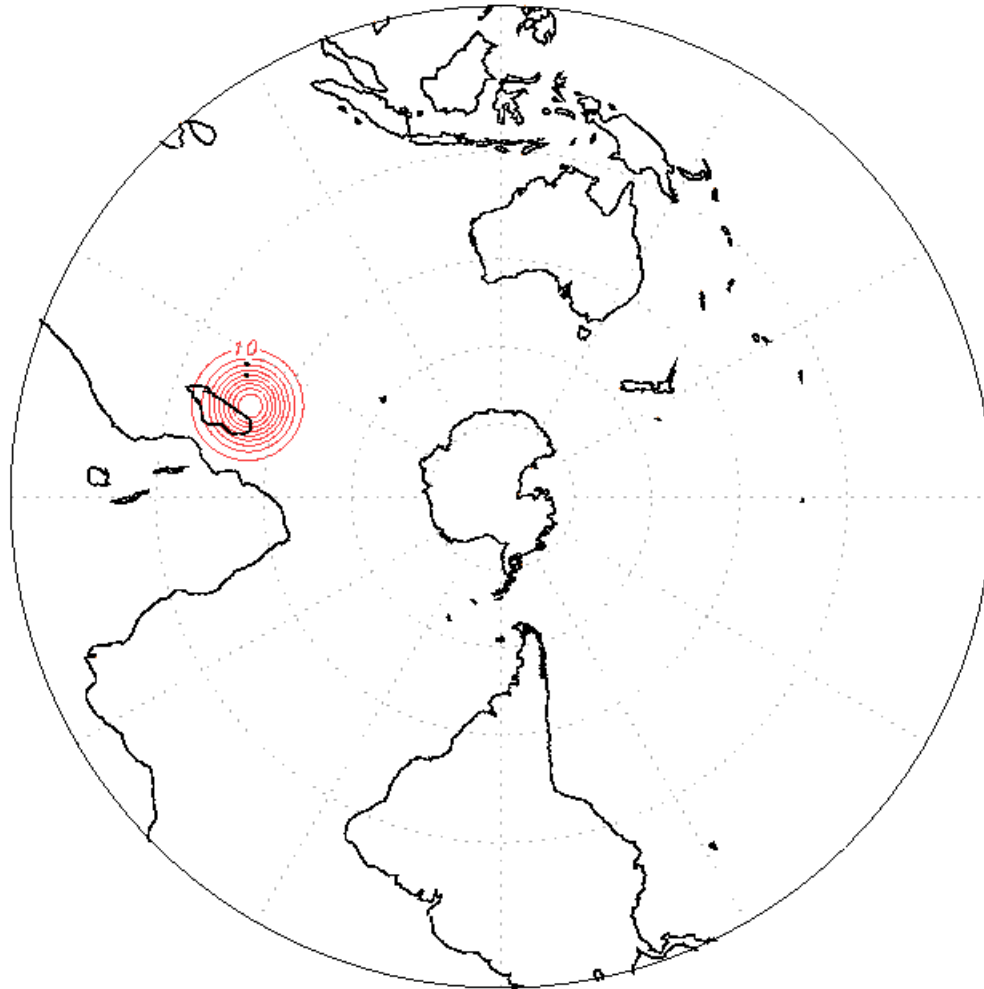
plot interval=10 with factor=1

h500 480hrs fcst (DATA_solid)



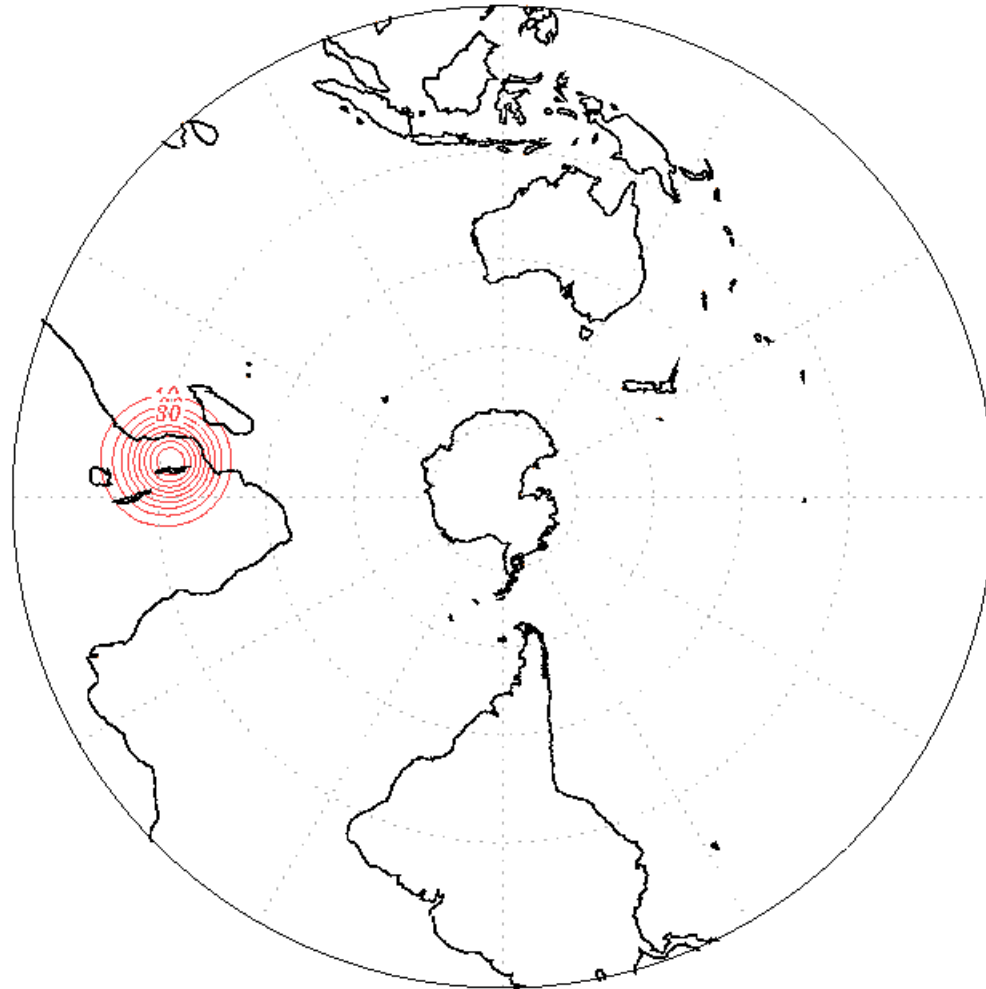
plot interval=10 with factor=1

h500 528hrs fcst (DATA_solid)



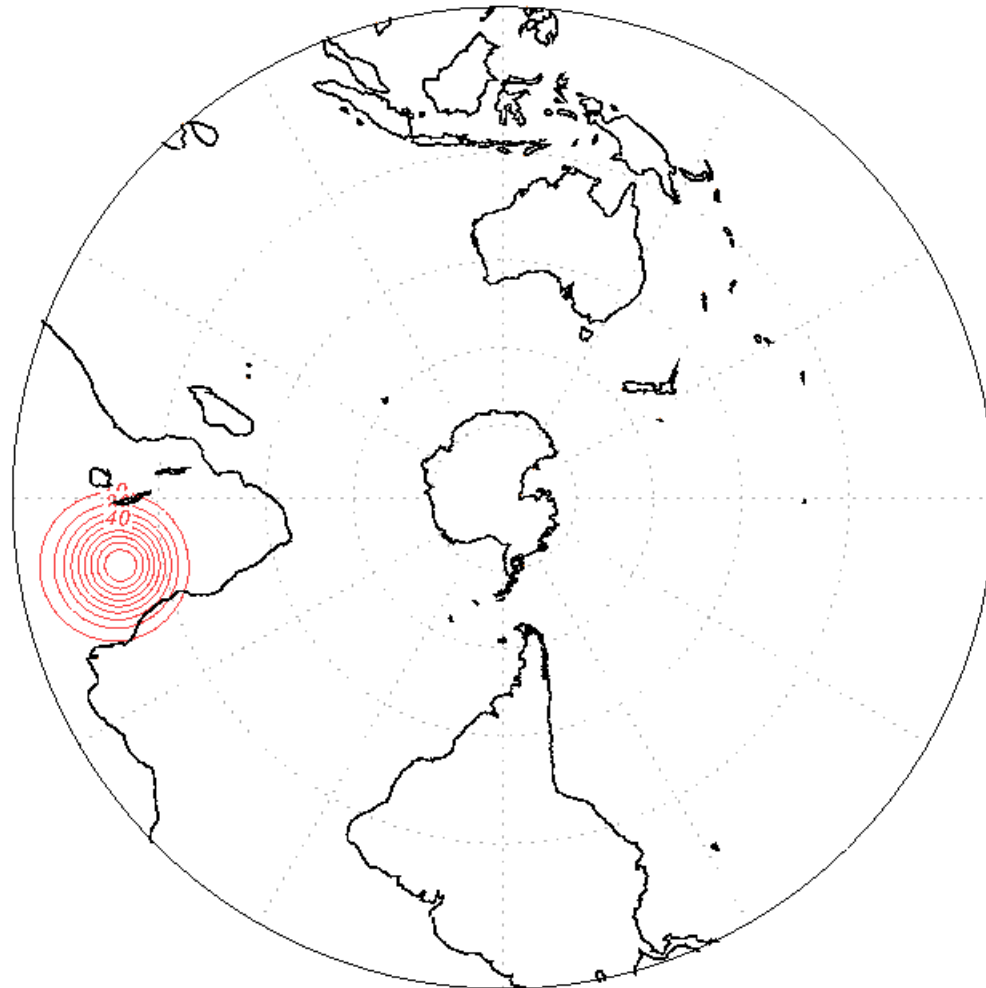
plot interval=10 with factor=1

h500 576hrs fcst (DATA_solid)



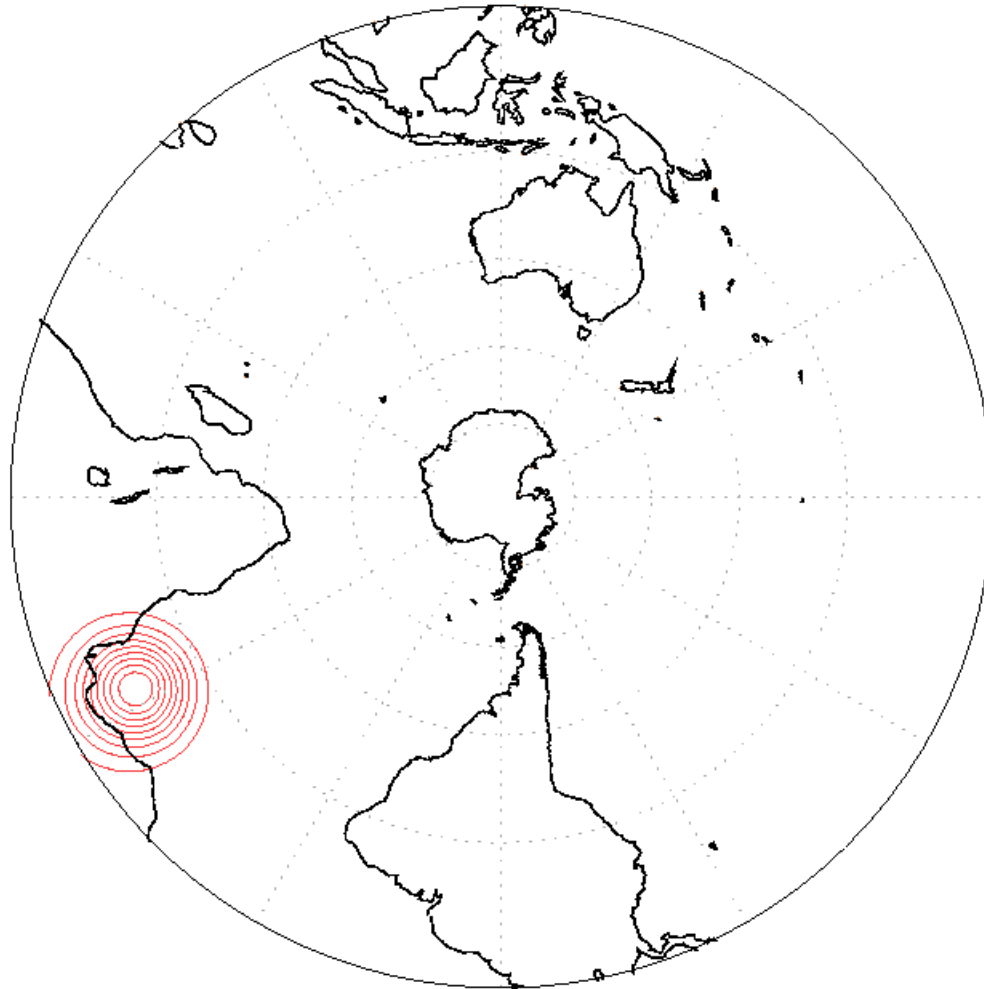
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h500 624hrs fcst (DATA_solid)



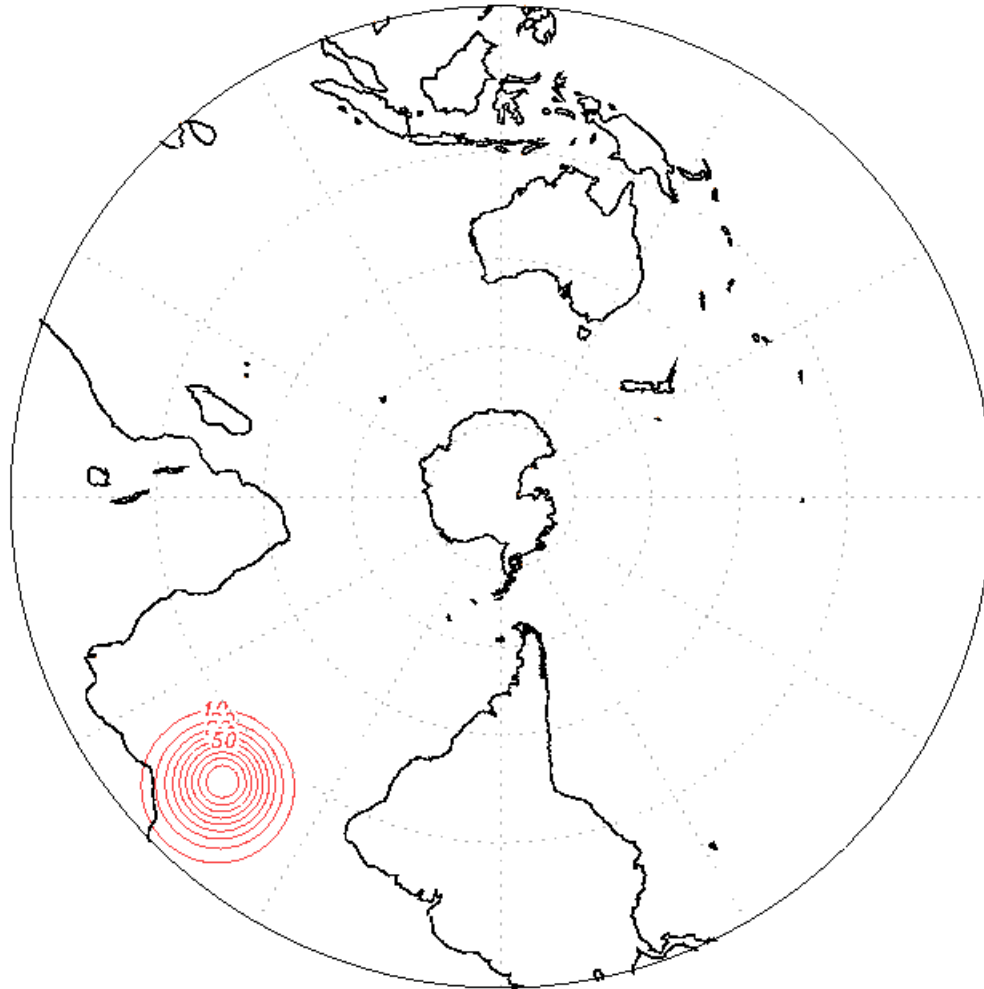
plot interval=10 with factor=1

h500 672hrs fcst (DATA_solid)



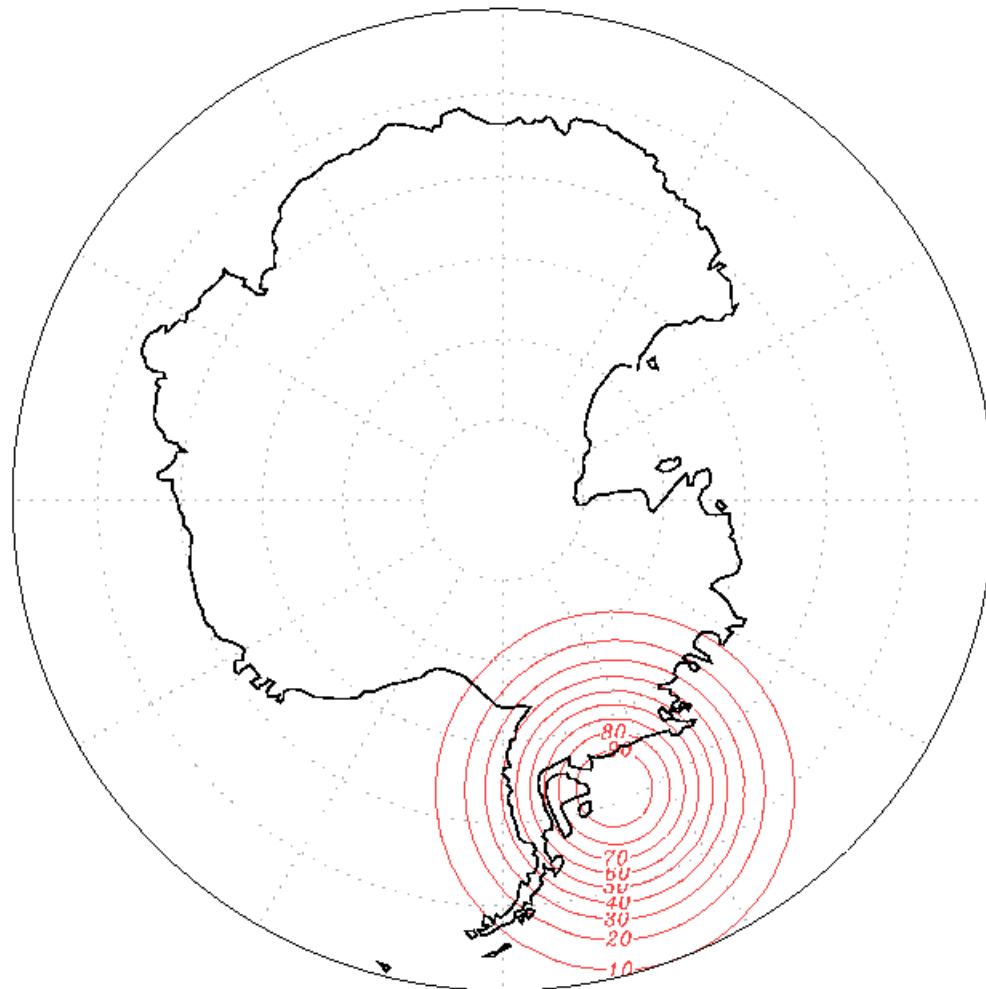
plot interval=10 with factor=1

h500 720hrs fcst (DATA_solid)



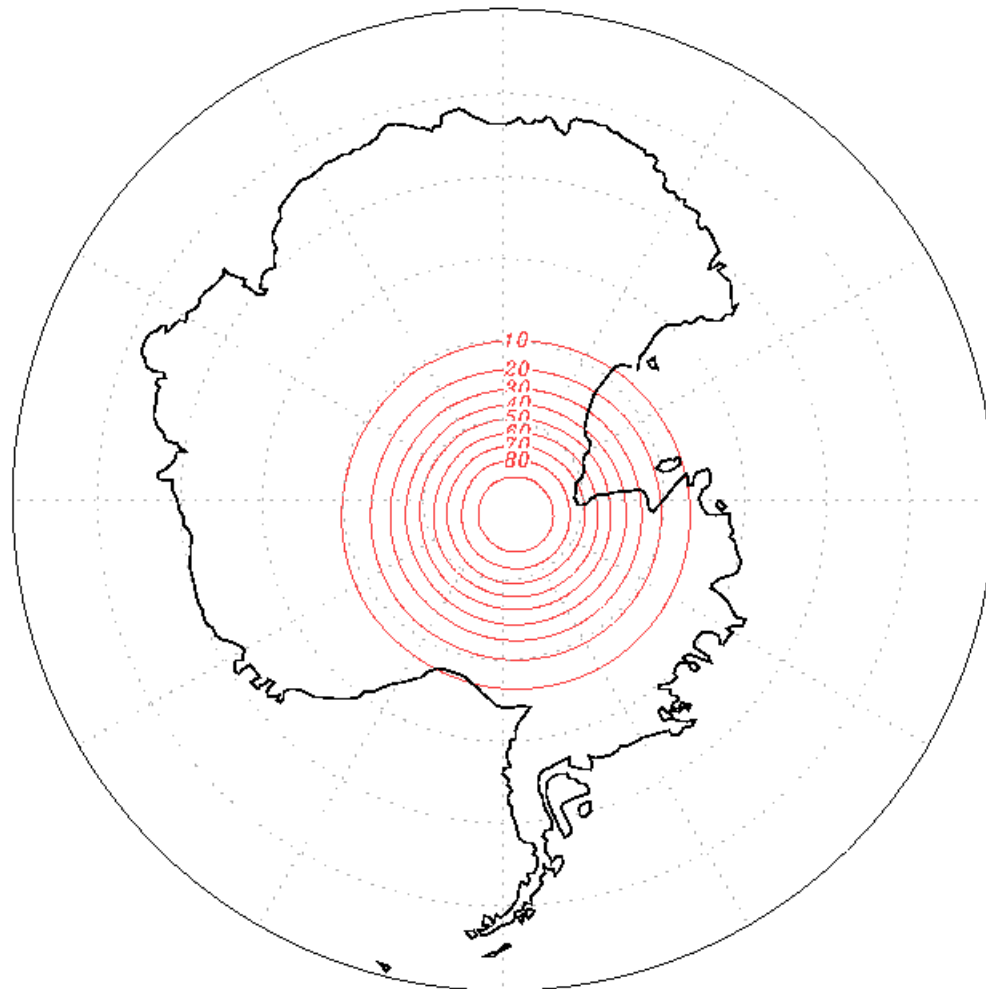
plot interval=10 with factor=1

h500 288hrs fcst (DATA_solid)



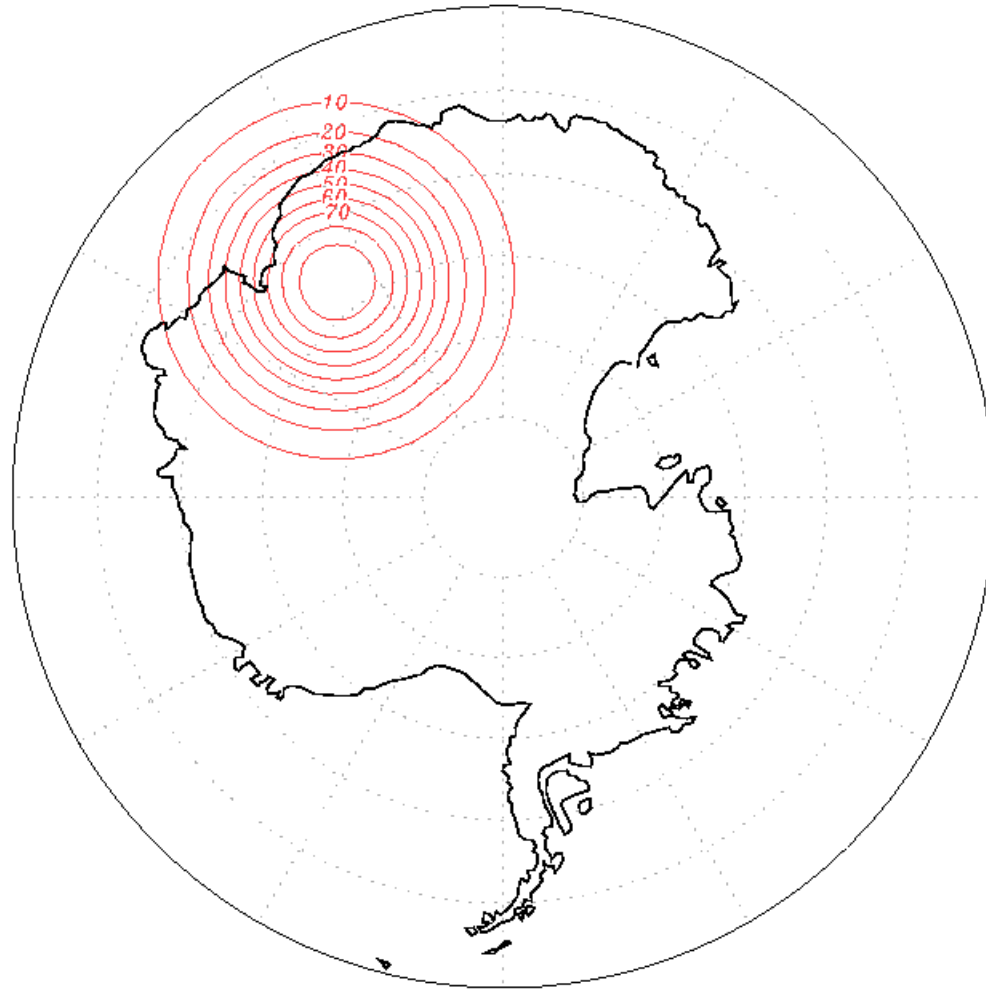
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h500 336hrs fcst (DATA_solid)



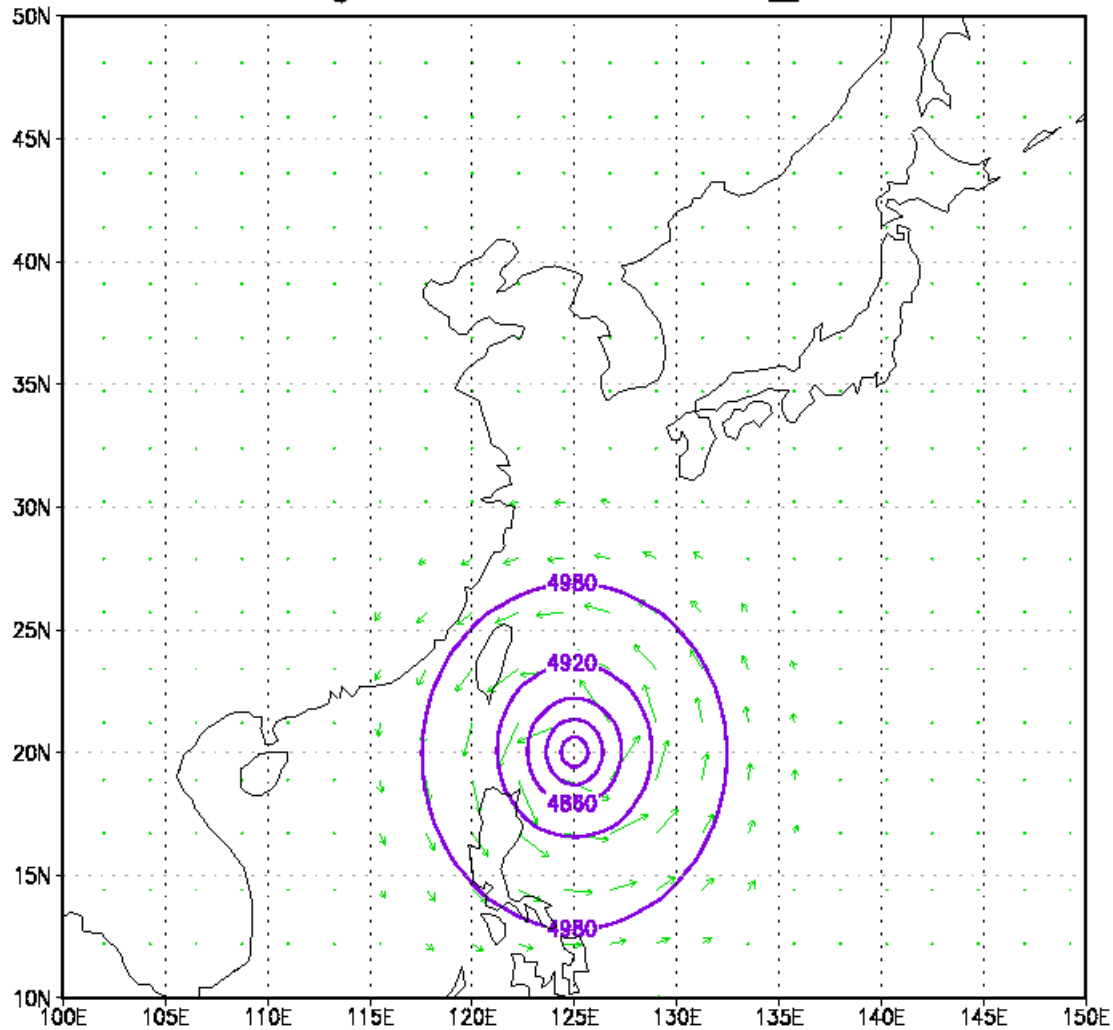
plot interval=10 with factor=1

h500 384hrs fcst (DATA_solid)

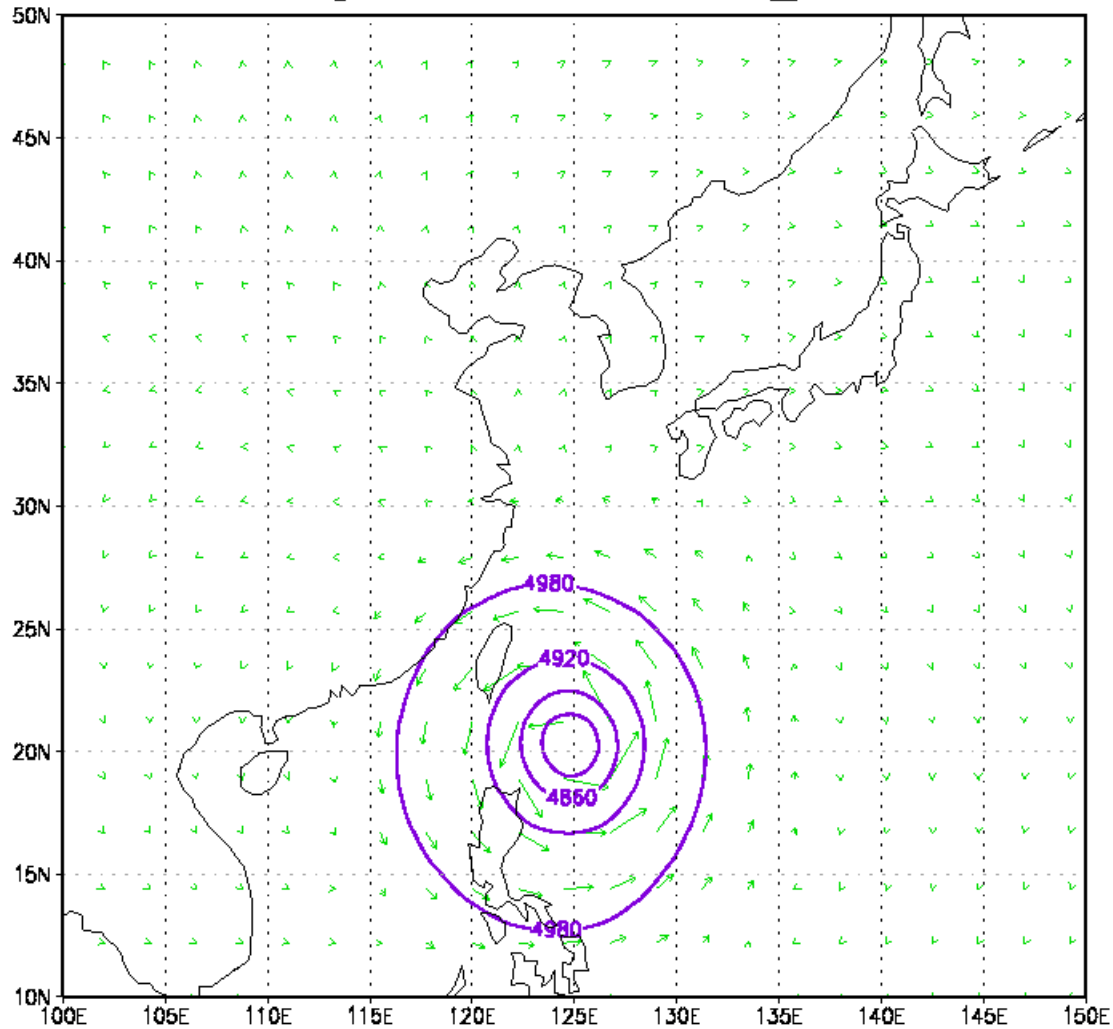


plot interval=10 with factor=1

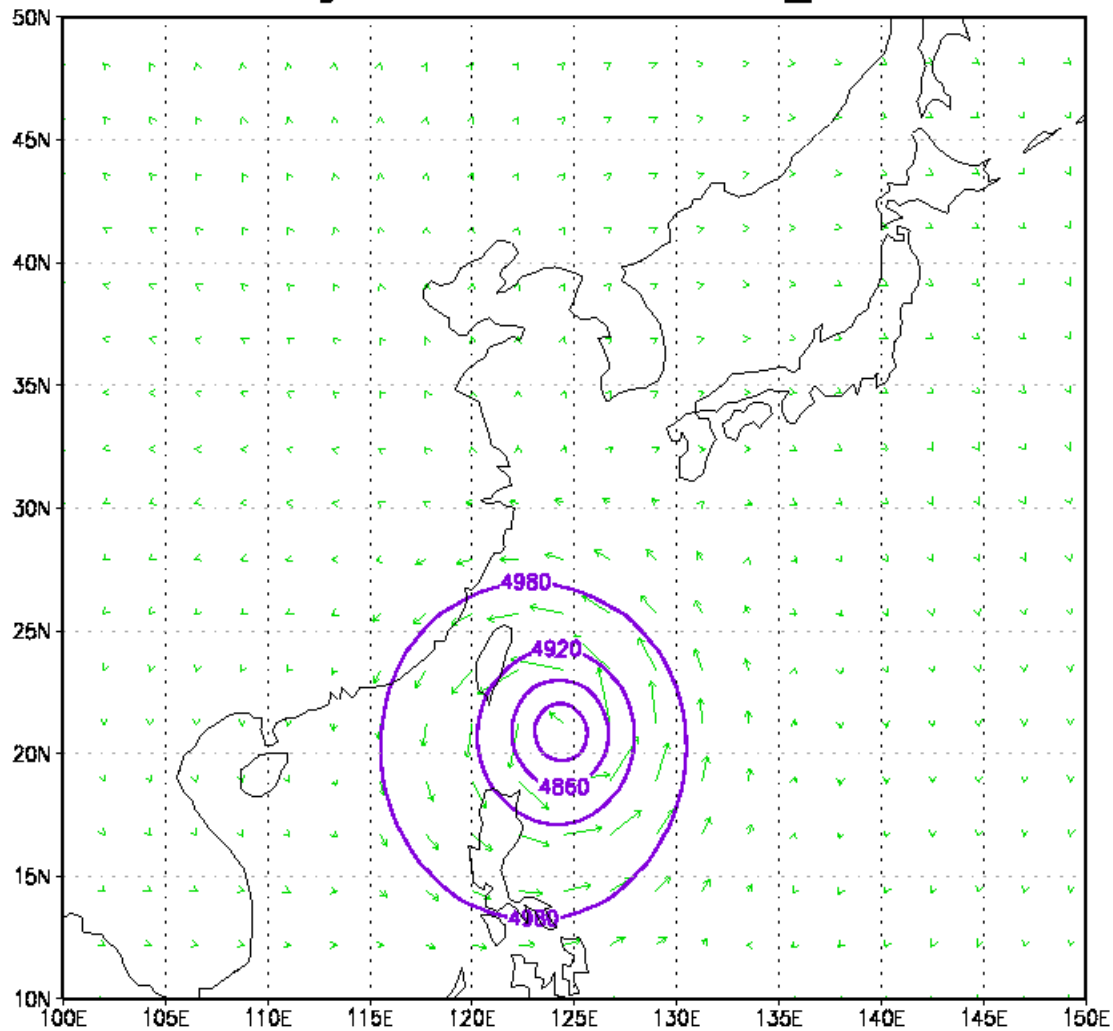
Height tau=0 - DATA_SISL



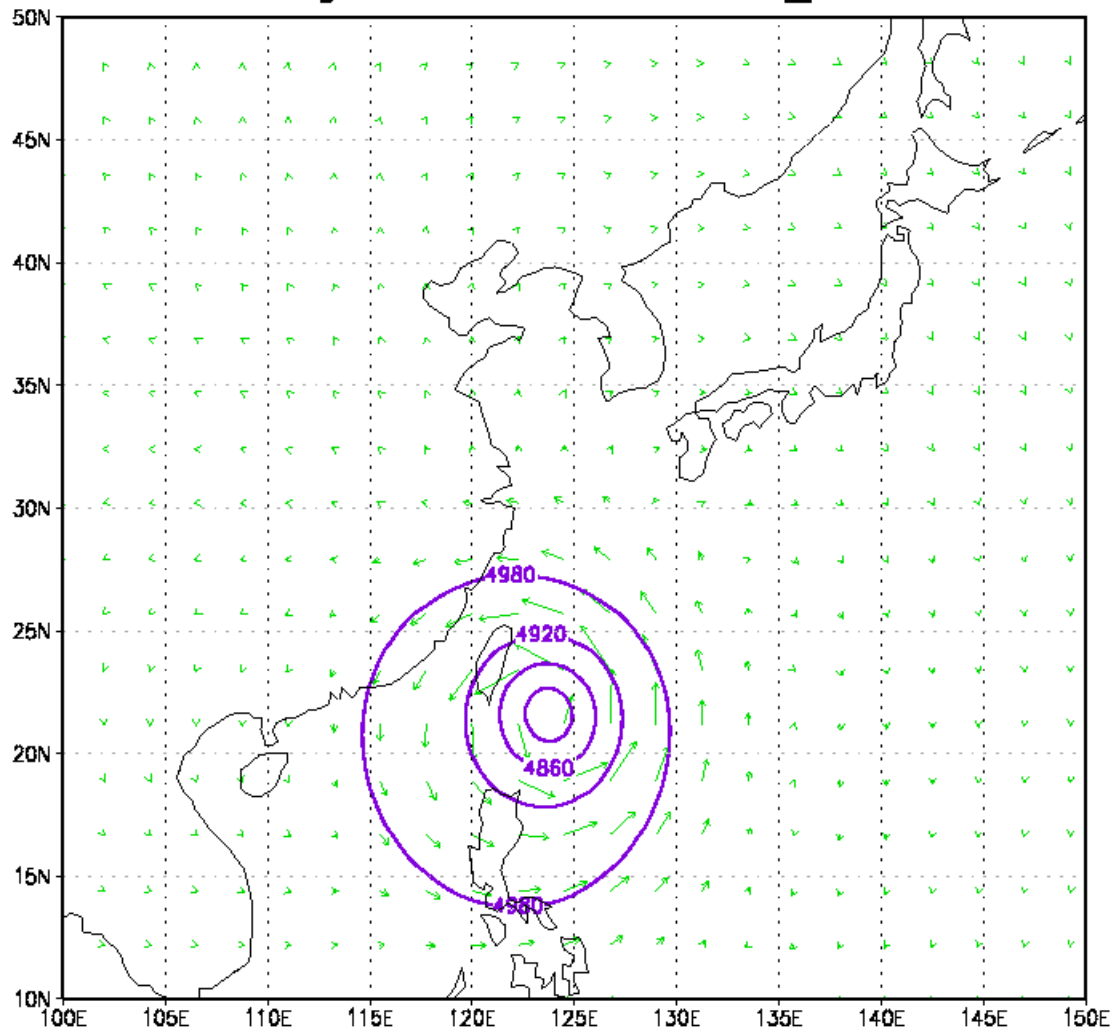
Height tau=6 - DATA_SISL



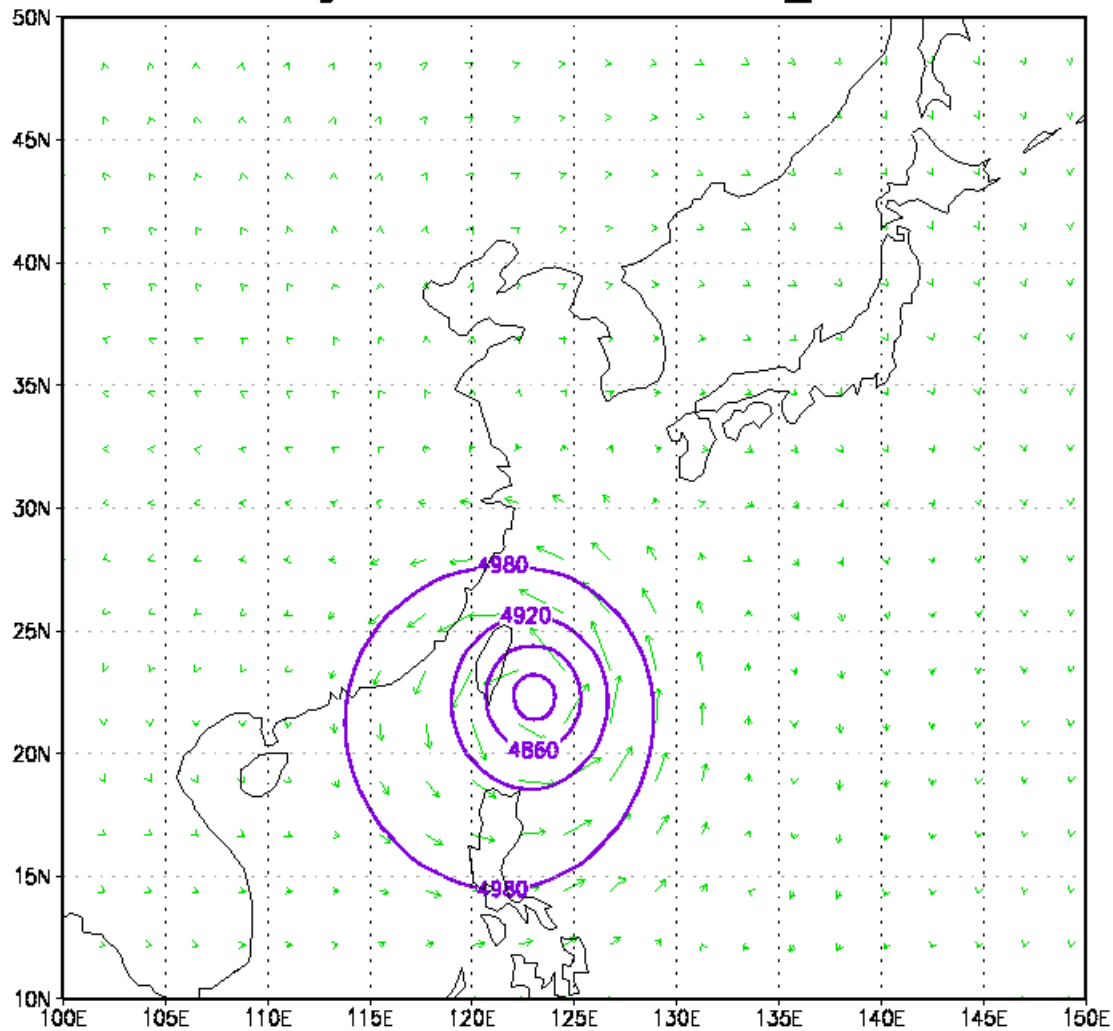
Height tau=12 - DATA_SISL



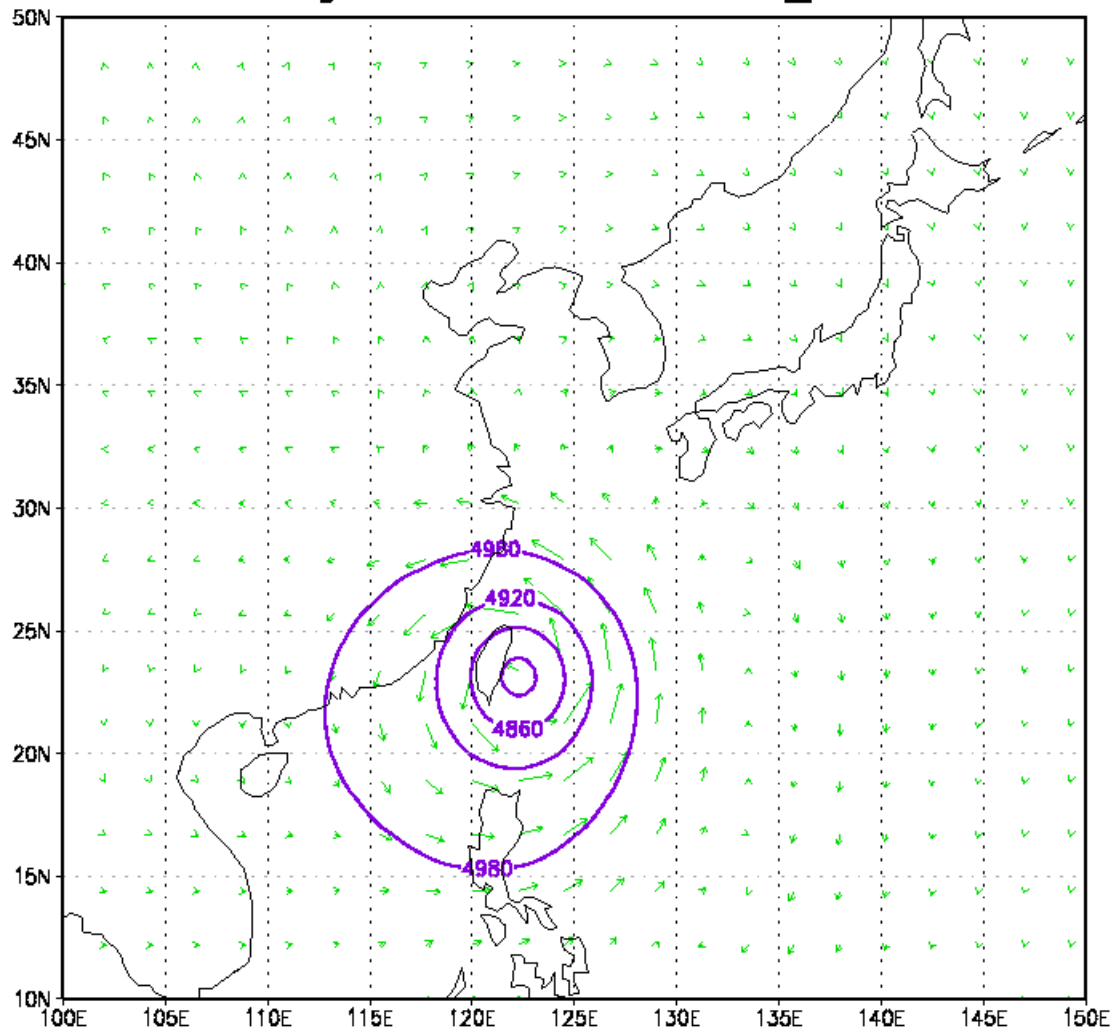
Height tau=18 - DATA_SISL



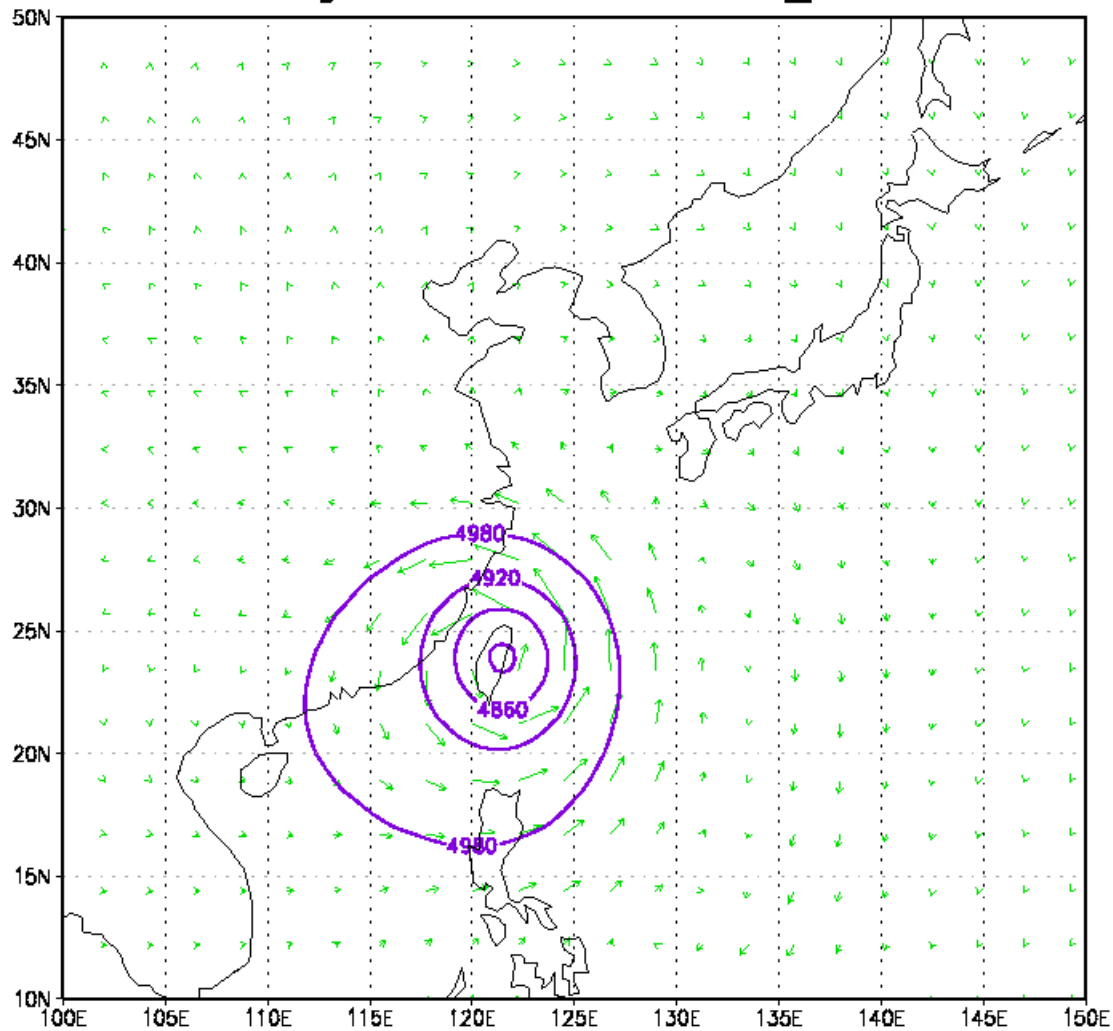
Height tau=24 - DATA_SISL



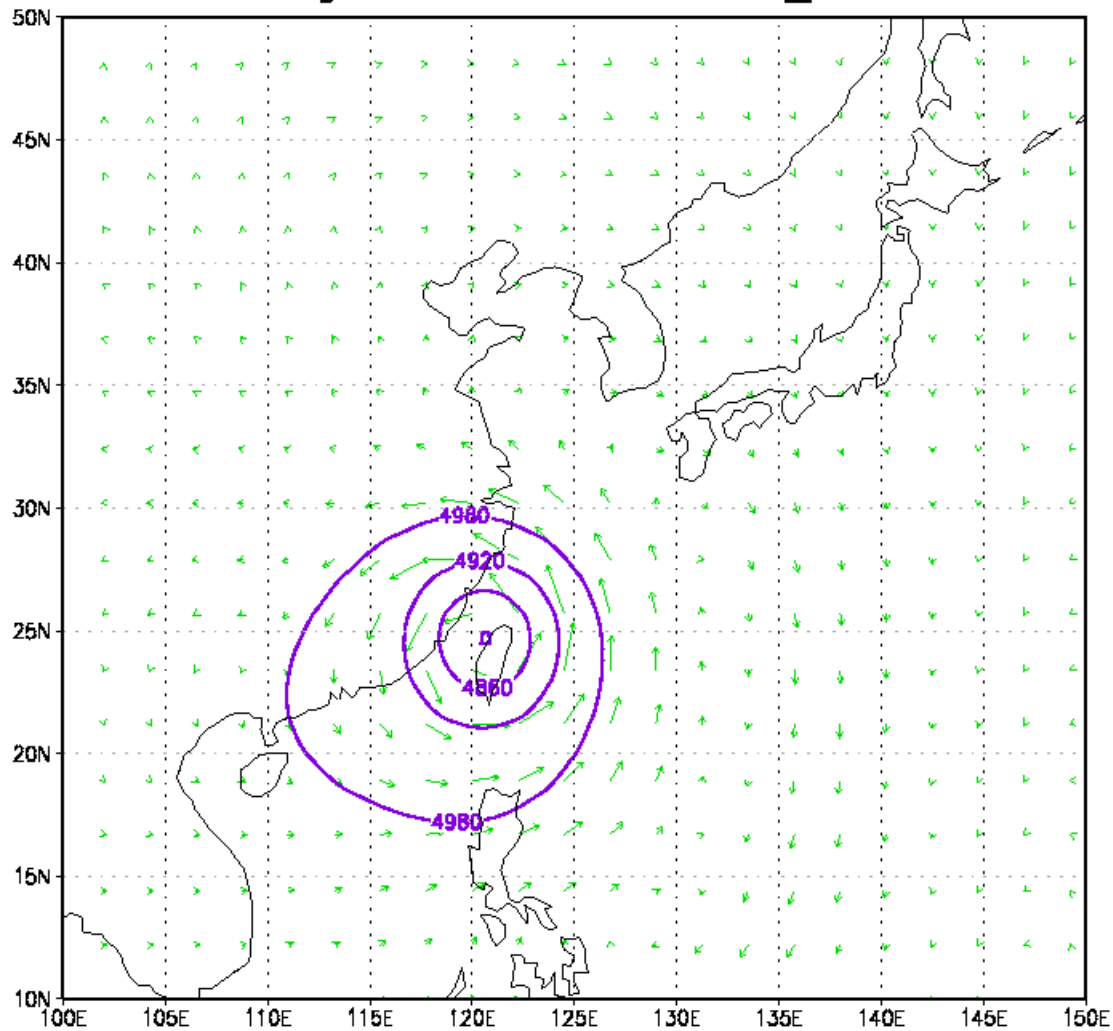
Height tau=30 - DATA_SISL



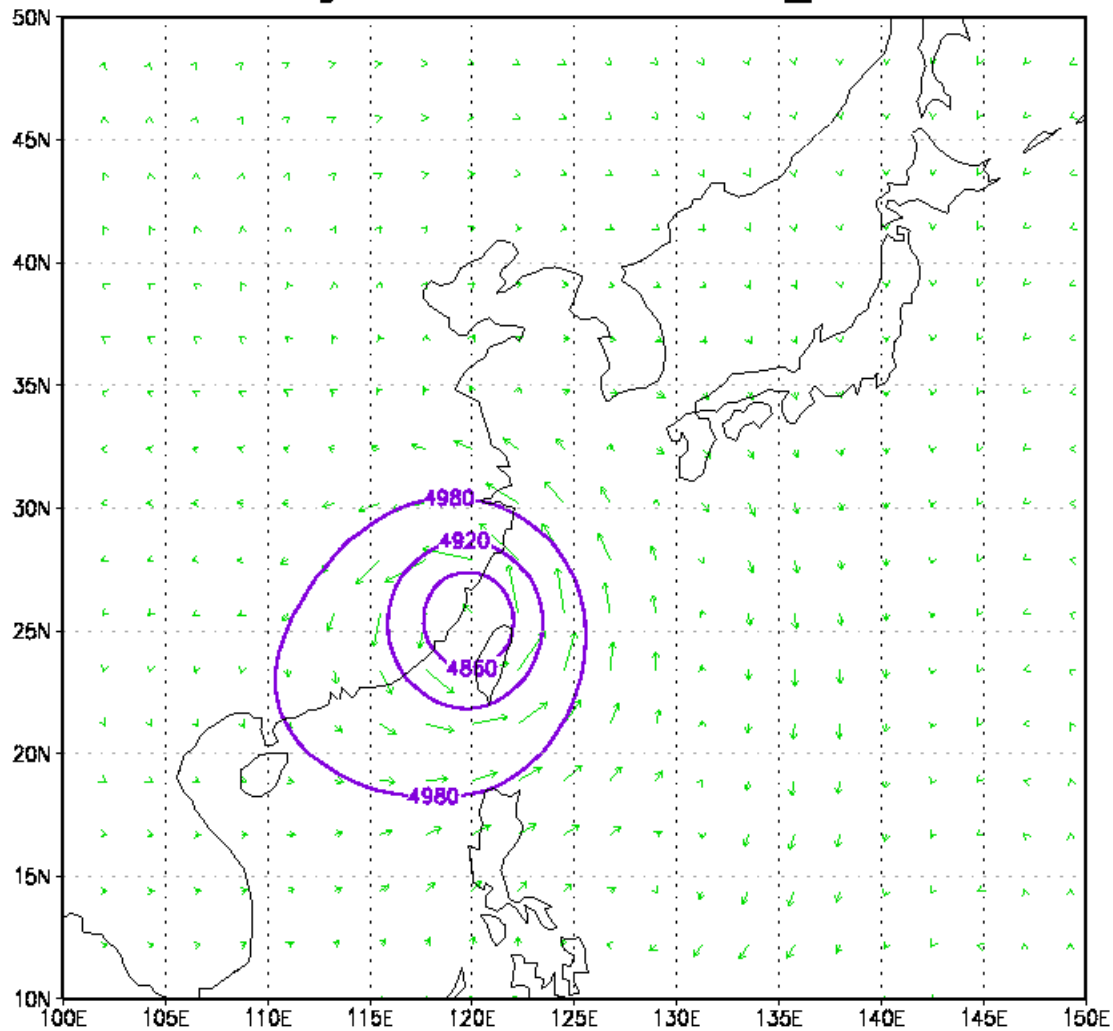
Height tau=36 - DATA_SISL



Height tau=42 - DATA_SISL



Height tau=48 - DATA_SISL



pressure tendency equation:

$$\frac{\partial \pi}{\partial t} = - \sum_{l=1}^L [\Delta A_l D_l + \Delta B_l M_l] = - \sum_{l=1}^L [\Delta p_l D_l + \Delta B_l \vec{V}_l \nabla \pi]$$

vertical motion:

$$\begin{aligned} \left[\dot{\eta} \frac{\partial p}{\partial \eta} \right]_{k+1/2} &= B_{k+1/2} \sum_{l=1}^L [\Delta A_l D_l + \Delta B_l M_l] - \sum_{l=1}^k [\Delta A_l D_l + \Delta B_l M_l] \\ &= B_{k+1/2} \sum_{l=1}^L [\Delta p_l D_l + \Delta B_l \vec{V}_l \nabla \pi] - \sum_{l=1}^k [\Delta p_l D_l + \Delta B_l \vec{V}_l \nabla \pi] \end{aligned}$$

momentum equation:

$$\begin{aligned} \frac{dU_k}{dt} &= fV_k - \left(\frac{1}{a^2} \frac{\partial \phi_k}{\partial \lambda} + \frac{c_p}{a^2} \theta_k \frac{\partial P_k}{\partial \pi} \frac{\partial \pi}{\partial \lambda} \right) \\ \frac{dV_k}{dt} &= -fU_k - \frac{\sin \varphi}{\cos^2 \varphi} (U_k^2 + V_k^2) - \left(\frac{\cos^2 \varphi}{a^2} \frac{\partial \phi_k}{\partial \mu} + \frac{\cos^2 \varphi}{a^2} c_p \theta \frac{\partial P_k}{\partial \pi} \frac{\partial \pi}{\partial \mu} \right) \end{aligned}$$

virtual potential temperature equation:

$$\frac{d\theta_k}{dt} = 0$$

moisture equation:

$$\frac{dq_k}{dt} = 0$$

$$P_k = \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \theta_j + \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\overline{\partial P_k}}{\partial \pi} \right) \pi$$

$$U_k^+ + \frac{\Delta t}{a^2} \beta \frac{\partial P_k^+}{\partial \lambda} = (Q_1)_k \quad + : (t + \Delta t)$$

$$V_k^+ + \frac{\Delta t \cos^2 \varphi}{a^2} \beta \frac{\partial P_k^+}{\partial \mu} = (Q_2)_k \quad 0 : t$$

$$\theta_k^+ + \Delta t \beta \sum_{l=1}^L s_{kl} D_k^+ = (Q_3)_k \quad - : (t - \Delta t)$$

$$q_k^+ = (Q_4)_k$$

$$\pi^+ + \Delta t \beta \sum_{k=1}^L \overline{\Delta p_k} D_k^+ = Q_5$$

$$Q_1 = U_k^- - \beta \Delta t \frac{1}{a^2} \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \frac{\partial \theta_j^-}{\partial \lambda} - \beta \Delta t \frac{1}{a^2} \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\overline{\partial P_k}}{\partial \pi} \right) \frac{\partial \pi^-}{\partial \lambda}$$

$$+ 2\Delta t \left(fV_k - \left(\frac{1}{a^2} \frac{\partial \phi_k}{\partial \lambda} + \frac{c_p}{a^2} \theta_k \frac{\partial P_k}{\partial \pi} \frac{\partial \pi}{\partial \lambda} \right)^0 \right) + 2\Delta t \beta \frac{1}{a^2} \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \frac{\partial \theta_j^0}{\partial \lambda}$$

$$+ 2\Delta t \beta \frac{1}{a^2} \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\overline{\partial P_k}}{\partial \pi} \right) \frac{\partial \pi^0}{\partial \lambda}$$

$$Q_2 = V_k^- - \beta \Delta t \frac{\cos^2 \varphi}{a^2} \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \frac{\partial \theta_j^-}{\partial \mu} - \beta \Delta t \frac{\cos^2 \varphi}{a^2} \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\overline{\partial P_k}}{\partial \pi} \right) \frac{\partial \pi^-}{\partial \mu}$$

$$+ 2\Delta t \left(-fU_k - \frac{\sin \varphi}{\cos^2 \varphi} (U_k^2 + V_k^2) - \frac{\cos^2 \varphi}{a^2} \left(\frac{\partial \phi_k}{\partial \mu} + c_p \theta_k \frac{\partial P_k}{\partial \pi} \frac{\partial \pi}{\partial \mu} \right)^0 \right)$$

$$+ 2\Delta t \beta \frac{\cos^2 \varphi}{a^2} \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \frac{\partial \theta_j^0}{\partial \mu} + 2\Delta t \beta \frac{\cos^2 \varphi}{a^2} \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\overline{\partial P_k}}{\partial \pi} \right) \frac{\partial \pi^0}{\partial \mu}$$

$$Q_3 = \theta_k^- - \Delta t \beta \sum_{l=1}^L s_{kl} D_k^- + 2\Delta t \beta \sum_{l=1}^L s_{kl} D_k^0$$

$$Q_4 = q^-$$

$$Q_5 = \pi^- - \Delta t \beta \sum_{k=1}^L \overline{\Delta p_k} D_k^- + 2\Delta t \left(\sum_{k=1}^L \Delta B_k \left[\left(\frac{\partial \pi}{\partial t} + \vec{V}_k \nabla \pi \right) \right]^0 \right) + 2\Delta t \beta \sum_{k=1}^L \overline{\Delta p_k} D_k^0$$

$$\zeta_k^+ = \alpha(Q_2, -Q_1)_k$$

$$D_k^+ + \Delta t \beta \nabla^2 P_k^+ = \alpha(Q_1, Q_2)_k$$

$$D_k^+ + \Delta t \beta \nabla^2 \left(\sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \theta_j^+ + \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\partial P_k}{\partial \pi} \right) \pi^+ \right) = \alpha(Q_1, Q_2)_k$$

$$D_k^+ + \Delta t \beta \nabla^2 \left(\sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} ((Q_3)_k - \Delta t \beta \sum_{l=1}^L s_{kl} D_k^+) + \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\partial P_k}{\partial \pi} \right) ((Q_5)_k - \Delta t \beta \sum_{k=1}^L \overline{\Delta p_k} D_k^+) \right) = \alpha(Q_1, Q_2)_k$$

$$\begin{aligned} D_k^+ - \beta^2 \Delta t^2 \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \sum_{l=1}^L s_{kl} \nabla^2 D_k^+ - \beta^2 \Delta t^2 \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\partial P_k}{\partial \pi} \right) \sum_{k=1}^L \overline{\Delta p_k} \nabla^2 D_k^+ \\ = \alpha(Q_1, Q_2)_k - \beta \Delta t \sum_{l=1}^L \sum_{j=1}^L a_{kl}^{-1} b_{lj} \nabla^2 (Q_3)_k - \beta \Delta t \left(\sum_{l=1}^L a_{kl}^{-1} c_l + c_p \bar{\theta}_k \frac{\partial P_k}{\partial \pi} \right) \nabla^2 (Q_5)_k \end{aligned}$$

三、結論



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謝謝